Solutions

Astrophysics Quiz Dec 2nd

$\sigma_T = 6.652\times10^{-29}\ m^2$, $m_p = 1.67\times10^{-27}$ kg , $M_\odot = 1.99\times10^{30}$ kg , $G = 6.67\times10^{-11}$, $I_{SP} = \frac{2}{5}MR^2$, $R_{NS} \simeq 10^4$ m

A. If the Force exerted by radiation pressure on a hydrogen atom, is

$F_{rad} = L\sigma_T/4\pi r^2 c$

Derive an expression for the maximum luminosity of an accretion powered neutron star or black hole.

Equate force of gravity on an infalling H atom with force of radiation going radially outward.

$$\frac{L\sigma_T}{4\pi R^2 c} = \frac{GM^* m_p}{R^2} \quad \therefore \quad L_{max} = \frac{GM^* m_p \times 4\pi c}{\sigma_T}$$

B. Calculate the energy gained by a neutron star X-ray pulsar, if its pulse period is observed to decrease from 75 seconds to 73 seconds over a period of one week.

NS mass $\approx 1.4-2\ M_\odot$. I picked $1.4 M_\odot$, $R_{NS} = 10^4$ m.

$$\Delta KE = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$$
$$= \frac{1}{2}\times\frac{2}{5}M_{NS}R_{NS}^2(\omega_2^2 - \omega_1^2)$$
$$= \frac{1}{5}M_{NS}R_{NS}^2\left(\frac{2\pi}{73} - \frac{2\pi}{75}\right)$$
$$= 2.23\times10^{34}\ J$$

Over a week, the implied ~~[illegible]~~ rate of energy gain is $\frac{\Delta KE}{Time} = \frac{2.23\times10^{34}}{7\times24\times60^2} = 3.7\times10^{28}$ W

C. Compare this result with the maximum accretion-powered luminosity for a 1.4 M_{sun} neutron star, and compute the maximum spin-up rate for a neutron star.

$$L_{max} = \frac{4\pi c\, GM^* m_p}{\sigma_T} = 1.8\times10^{31}\ W$$

$(\simeq 10^{38}$ erg/s$)$

So required accretion rate is 1000X smaller than Eddington limit.

Maximum rate when Energy gain $\approx L_{max}$

$$\frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = L_{max}$$
$$\frac{1}{2}I\frac{d(\omega^2)}{dt}$$
$$\frac{1}{2}I\times2\omega\frac{d\omega}{dt} = L_{max}$$

$$\therefore \dot\omega_{max} = \frac{L_{max}}{I\omega} \simeq 1.88\times10^{-6}\ rad\,s^{-2}$$

(note observed $\dot\omega = 3.8\times10^{-9}$)

D. What is the name of this maximum luminosity condition, and how does it depend on the composition of the infalling gas?

The Eddington Limit. It depends on the cross-section for absorption of the gas atoms. Hence higher opacity lowers the limit. Opacity originates in chemical enrichment (absorption lines).