Homework Solutions

① Rate of H → He in the sun kg/s and $M_\odot/yr$

$$L_{sun} = 3.839 \times 10^{26} \text{ watts}$$

$$E = mc^2$$

$$m = \frac{E}{c^2} = \frac{3.839\times10^{26}}{(3\times10^8)^2} = 4.26\times10^9 \text{ kg/s}$$

$$M_{sun} = 1.99\times10^{30} \text{ kg} \longrightarrow \frac{4.26\times10^9}{1.99\times10^{30}} \times 365\times24\times60\times60$$

(This calculation ignores neutrino losses)

$$\therefore \frac{dM}{dt} = 6.76\times10^{-14} \ M_\odot/yr.$$

②a) Lifetime of $100M_\odot$ star with $L_* = 10^6 L_{sun}$.
Assume P-P chain only.

$$\text{Initial } N_{protons} \approx \frac{100\times2\times10^{30}}{M_{Hydrogen}} = \frac{1.99\times10^{32} \text{ kg}}{1.67\times10^{-27} \text{ kg}} = 1.189\times10^{59}$$

"Efficiency": P-P chain converts $4\,{}_1P \rightarrow {}_4He$

Each He nucleus brings about release of 26.73 MeV

so total energy available from P-P chain $\sim \frac{N_P}{4}\times 26.73 \text{ MeV}$

$$= 1.27\times10^{44} \text{ Joule}$$

$$T = \frac{E_{Total}}{L_*} = \frac{1.27\times10^{47}}{10^6\times3.839\times10^{26}} = 3.3\times10^{14} \text{ sec}$$

$$T = 1.05\times10^7 \text{ years.}$$

(2B) Dwarf Star $\frac{1}{2} M_\odot$, $\frac{L_\odot}{10}$

$$E_{Total} \simeq \frac{0.5 \times 1.99 \times 10^{30}}{4(1.67 \times 10^{-27})} \times 26.73 \text{ MeV}$$

$$\simeq 1.4895 \times 10^{56} \times 26.73 \times 10^{6} \times 1.6 \times 10^{-19}$$

$$\simeq 6.37 \times 10^{44}$$

$$T \sim \frac{E}{L} \simeq \frac{6.37 \times 10^{44}}{0.1 \times 3.89 \times 10^{26}}$$

$$\simeq 1.6 \times 10^{19} \text{ sec}$$

$$\tau \simeq 5.19 \times 10^{11} \text{ yrs.}$$

(Slightly different answers result if one discounts the energy from $e^- + e^+$ annihilation and neutrinos)

(3) How long could Kelvin Helmholtz contraction power a Brown Dwarf Star?

$$T_{KH} = \frac{E_{Grav}}{L_*} \qquad E_{Grav} \simeq \frac{GM^2}{R}$$

Mass and radius of a brown dwarf

$M_{BD} \sim 0.07 M_\odot$

$R_{BD} \sim R_{Jupiter} \sim$

$L_{BD} \sim 0.01\% L_\odot \sim 10^{-4} L_\odot$

Thus $E_{Grav} \sim \frac{6.67 \times 10^{-11} \times (0.07 \times 2 \times 10^{30})^2}{70 \times 10^6 \text{ m}}$

$\therefore \sim 1.86 \times 10^{42}$ J

$$T_{KH} \simeq \frac{E_G}{L_{BD}} \sim \frac{1.86 \times 10^{42}}{10^{-4} \times 3.89 \times 10^{26}} \simeq 4.8 \times 10^{19} \text{ sec} \simeq 1.5 \times 10^{12} \text{ yrs}$$

(4) Derive an Expression for the maximum Mass of a Star.
(Use Maoz Ch3, Q3 as a guide)

Because of Radiation pressure the most massive Stars are those where radiation pressure P_{rad} approximately equals the non-relativistic thermal pressure.

Condition for Stability : Virial theorem

$$-U_G = 2U_{Thermal}$$

$$\text{or} \quad \overline{P_{th}} = -\frac{1}{3}\frac{E_G}{V}$$

If $P_{Rad} \sim P_{Thermal}$ then gravity is barely able to keep the Star bound. $-U_G \approx U_T + U_{rad}$.

From Maoz 3a : $P \sim \left(\frac{4\pi}{3^4}\right)^{1/3} GM^{2/3}\rho^{4/3}$

from Maoz 3b : when $P_{rad} = P_{kinetic}$

$$P = 2\left(\frac{3}{a}\right)^{1/3}\left(\frac{k\rho}{\bar{m}}\right)^{4/3}$$

Thus: $\left(\frac{4\pi}{3^4}\right)^{1/3} GM^{2/3}\rho^{4/3} = 2\left(\frac{3}{a}\right)^{1/3}\left(\frac{k\rho}{m}\right)^{4/3}$

$$\therefore \quad M_{max} = \left(\frac{2}{G}\right)^{2/3}\left(\frac{3^5}{4\pi a}\right)^{1/2}\left(\frac{k}{m}\right)^2$$

$$M_{max} = 112 \ M_\odot$$