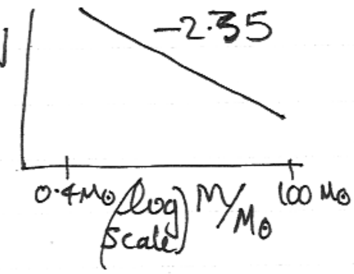
HW Solution   Maoz Ch5, P3 (a-3)

(a) $5\times10^{10}\,M_\odot$ Gas, 10^{11} Stars

Initial mass fn. $\frac{dN}{dM} \propto M^{-2.35}$



How many stars are formed above $8\,M_\odot$ and hence have gone SN ?

$$f(m>8) = \frac{\int_8^{100} m^{-2.35}\,dm}{\int_{0.4}^{100} m^{-2.35}\,dm} = \frac{\left[m^{-1.35}\right]_8^{100}}{\left[m^{-1.35}\right]_{0.4}^{100}}$$

$$= \frac{100^{-1.35} - 8^{-1.35}}{0.4^{-1.35} - 100^{-1.35}} = 0.017$$

Thus the total number of such stars is $0.017\times10^{+11} = 1.7\times10^{9}$

The lifetime of stars $>8M_\odot$ is short, much less than the age of our Sun. Hence the gas from which the Sun is made, should have been pre-enriched by the supernova in the preceding generation(s) of massive stars.

(b) Predicted Fe/H :

$$= \frac{N_{supernovae} \times \text{Mass of Iron per SN}}{\text{Gas mass in Galaxy}}$$

$$= \frac{1.7\times10^{9} \times 0.05\,M_\odot}{5\times10^{10}\,M_\odot}$$

$$Fe/H = 0.0017$$

Remarkably close to the actual measured value for the sun. 0.00177.

(c) How many Massive star binaries have formed, where <u>Both</u> stars are initially $> 8M_\odot$?

$$P(M>8M_\odot) = 0.017 \quad \text{and} \quad P_{Binary} = \frac{1}{2}$$

So probability of drawing <u>two</u> such stars in a binary is:

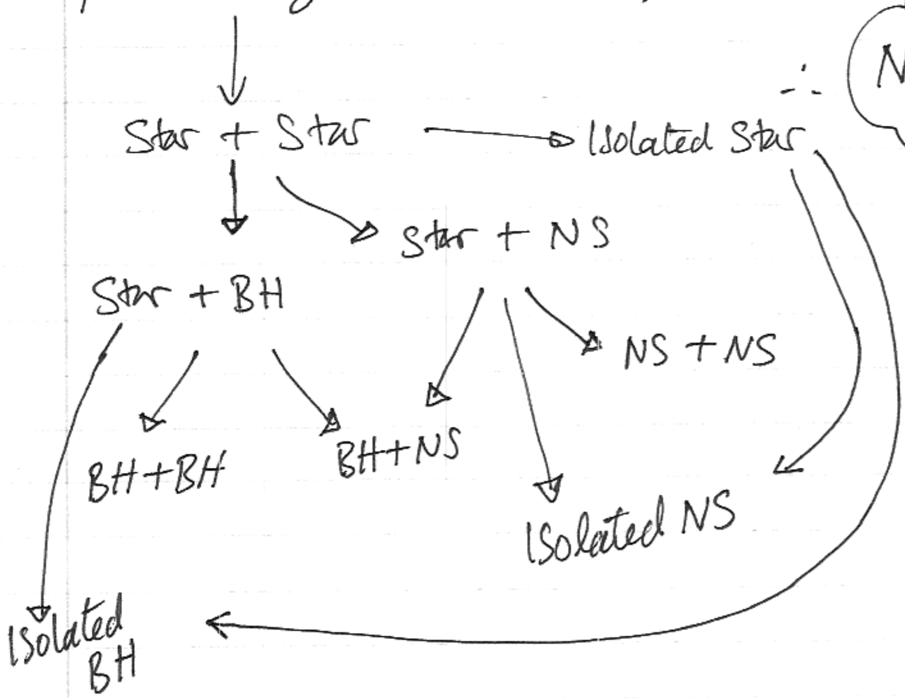
$$P_{Binary} \times P_8 \times P_8 = \frac{P^2}{2} = 0.0001445 = 1.445\times10^{-4}$$

Total number of such <u>Systems</u> is: $\dfrac{N_{stars}\times 1.445\times10^{-4}}{2}$

each binary consists of 2 stars!

$$\therefore \boxed{N_{\substack{\text{massive}\\ \text{Binary}}} = 7.2\times10^{6}.}$$

These massive binaries will eventually pass through several stages:



In my diagram, I have ignored binary interaction. for example the paths that lead to one star losing mass, so that it never reaches SN.
Also ignored binary survival at supernova (kick velocity)

(d) Assymetry in supernova explosion produces a 'kick' 500k
Clearly, if the kick velocity exceeds a critical value a binary system will not remain bound.

Equate Gravitational binding energy for two stars (point masses) seperated by a distance R.

Then solve for R_{max} such that $V_{critical} = 500$ km/s

limiting cases, both stars have $M = 8M_\odot$

$$(8M_\odot) \longleftarrow R \longrightarrow (8M_\odot)$$

$$\therefore \quad U_G = \frac{GM_1 M_2}{R}$$

$$U_G = \frac{8^2 G M_\odot^2}{R} = \frac{64 G M_\odot^2}{R}$$

Now the kinetic energy of the newly born Neutron Star is

$$KE_{NS} = \frac{1}{2} M_{NS} V_{kick}^2$$

Thus for critical case. $KE = U_G$

$$\frac{1.44}{2} \cancel{M_\odot} V_c^2 = \frac{8^2 G M_\odot^{\cancel{2}}}{R}$$

$$R = \frac{2 \times 64\, GM_\odot}{1.4 (500\,\text{km/s})^2}$$

$$= 4.8 \times 10^{10} \text{ m}$$

$1\,AU = 150 \times 10^6$ km
$= 1.5 \times 10$