Geotechnical Engineering Research Laboratory
Dept. of Civil and Environmental Engineering
University of Massachusetts Lowell.

Reliability Analysis for the ULS of Shallow Foundations

14.533 Advanced Foundation Engineering

The lecture is based on

NCHRP Report 651

LRFD DESIGN AND CONSTRUCTION OF SHALLOW FOUNDATIONS FOR HIGHWAY STRUCTURES

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OUTLINE

- **Background**
  1. Objectives
  2. Method of Approach
- **Databases**
  - Database Summary
  - Database Flow Chart
- **Design & Construction Practices – Questionnaire**
- **BC of Shallow Foundations on Soil**
  1. Determination of ULS from Case Histories
  2. Failure (Ultimate Load) Criteria
  3. Uncertainty Evaluation
     - BC of Centric Vertically Loaded Footing on Granular Soils
     - BC of eccentric Vertically Loaded Footing on Granular Soils
     - BC of inclined Loaded Footing on Granular Soils
  4. Calibration of Resistance Factors
  5. Example
  6. Summary and Conclusions
- **BC of Shallow Foundations on Rock**
  1. Broad Objectives
  2. Database UML/GTR RockFound07
  3. Rock Classification and Properties
  4. Methods of Analyses Selected for Establishing the Uncertainty in B.C. of Foundations on Rock
  5. Calibration of resistance factors
  6. Summary and Conclusions
- **General Conclusions and Recommendations**
- **Summary**

OBJECTIVES
NCHRP RESEARCH PROJECT 24-31

Develop and Calibrate Procedures and Modify AASHTO's Section 10 (Foundations) Specifications for the Strength Limit State Design of Bridge Shallow Foundations.

For NCHRP Research Report 651, Google NCHRP 651

Method of Approach

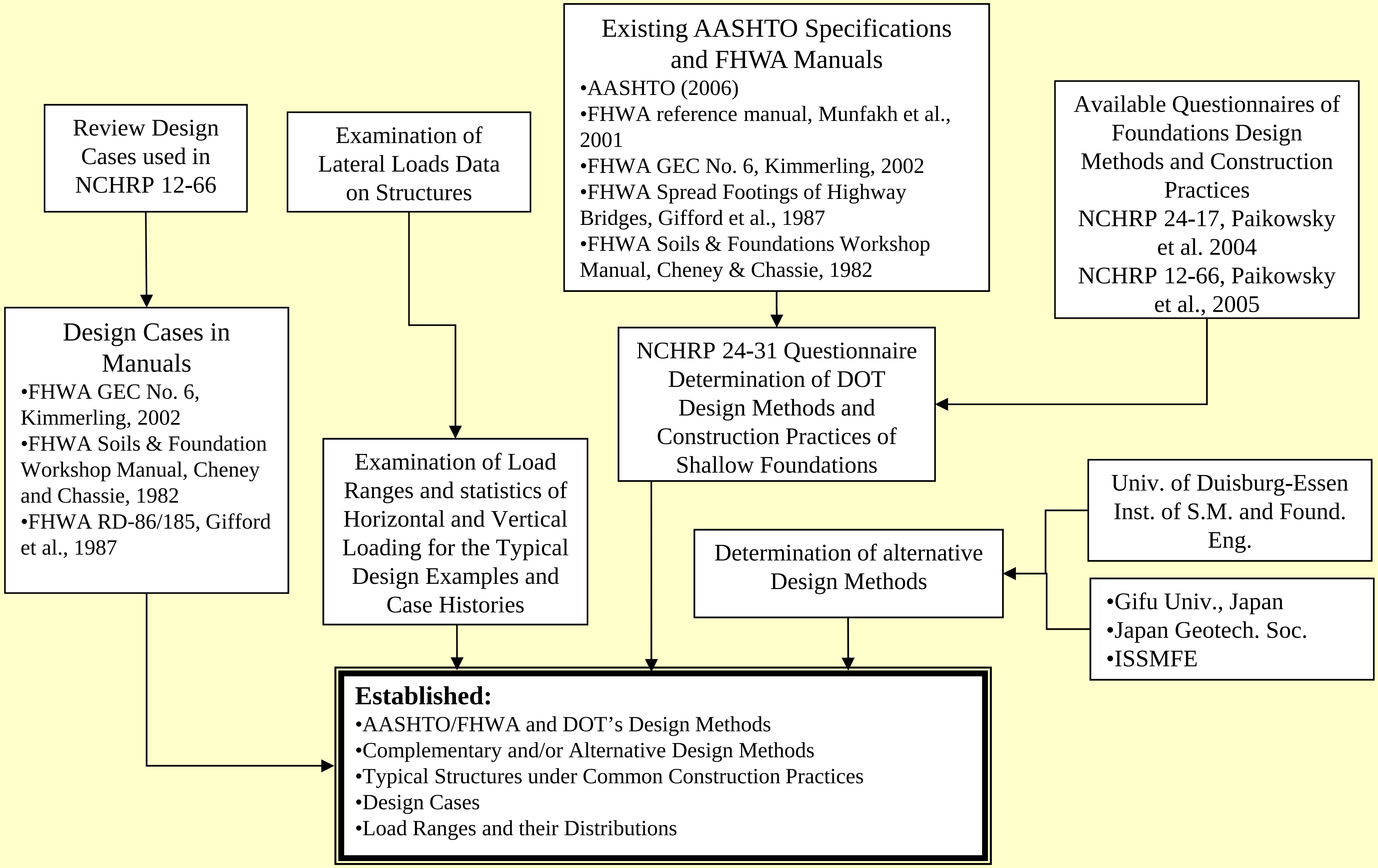


Figure 41. Flowchart outlining the research plan for Unit I(a) establishing design methods, construction practices, design cases, and loads.

Method of Approach

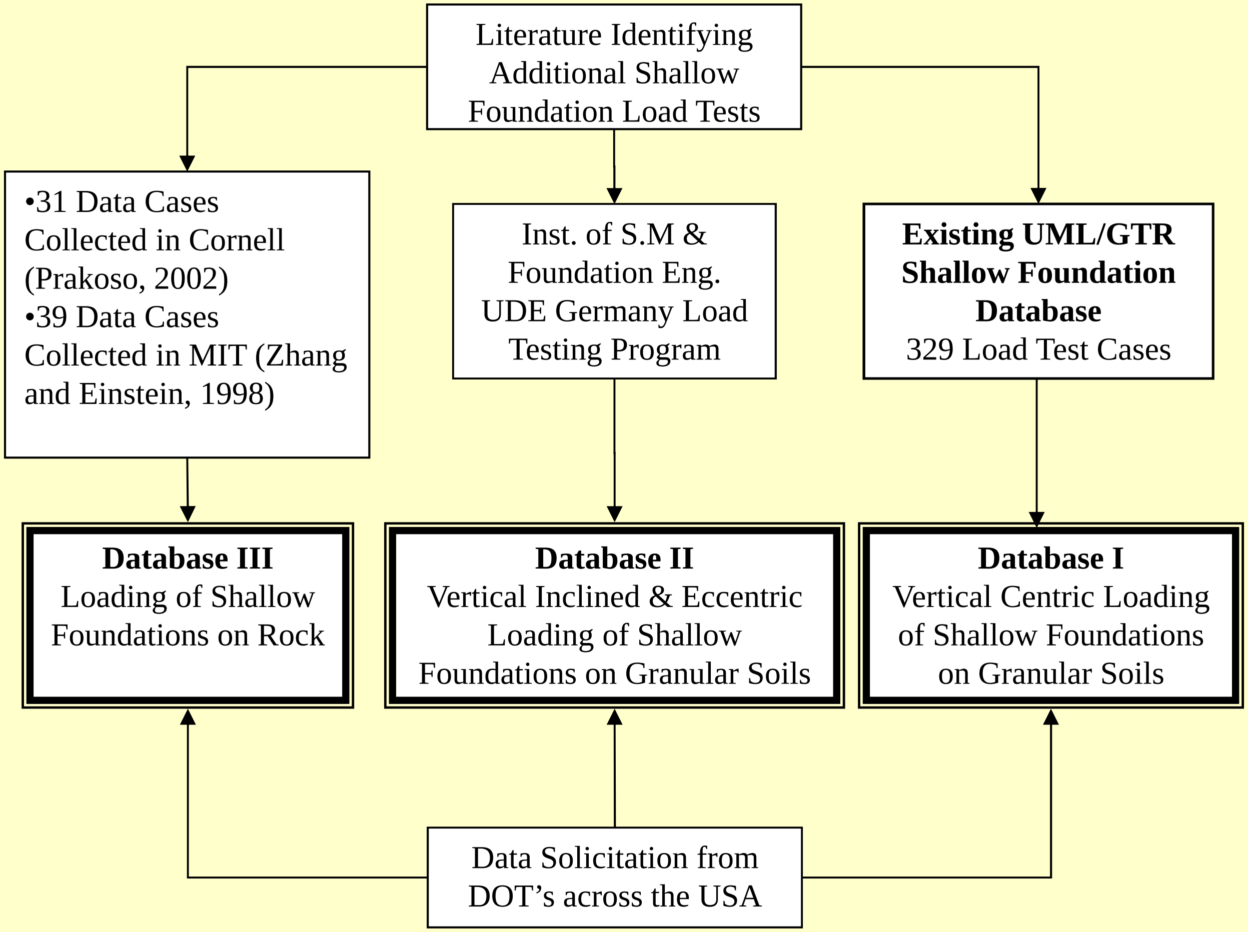


Figure 42. Flowchart outlining the research plan for Unit I(b) – establishing databases for shallow foundations load tests.

Design & Construction Practices Questionnaire

- **Developed and distributed to 161 State Highway Officials, TRB Representatives, and State and FHWA bridge engineers.**
- **Obtained responses from 39 states and 1 Canadian Province**
- **Previous relevant information was obtained via a questionnaire circulated in 2004 for the research project *NCHRP 12-66 AASHTO LRFD Specifications for the Serviceability in the Design of Bridge Foundation***

Design & Construction Practices - Questionnaire

Foundation Alternatives

Results on distribution of bridge foundation usage from our previous questionnaires conducted in 1999 and 2004, and the current questionnaire (over the past 3 years, 2004-2006):

	shallow foundations	*driven piles*	*drilled foundations*
1999/2004	***14%/17%***	***75%/62%***	***11%/21%***
current	***17%***	***59%***	***24%***

The use of shallow foundations was not changed overall relative to the last survey (2004). There is a consistent trend, however, in the decrease of the use of driven piles (75%, 62%, and 59% for 1999, 2004, and 2007, respectively) and increase of the use of drilled foundations (11%, 21%, and 24% for 1999, 2004, and 2007, respectively).

There is some discrepancy between the total foundation use and the percentage of use specifically addressing piers and abutments. Some of this discrepancy can be attributed to the fact that all foundations include non-bridge structures like buildings, posts and sound barriers.

The average use presented above, changes significantly across the country. The presented number that relates to bridge foundations only (with average use of 17.7% for abutments and piers).

The use of shallow foundations in the Northeast exceeds by far all other regions of the USA, ranging from 40% in NY, NJ and ME, to 67% in CT. Other "heavy users" are TN (63%), WA (30%), NV (25%) and ID (20%). In contrast, out of the 39 responding states, six states do not use shallow foundations for bridges at all, and additional eight states use shallow foundations in 5% or less of the highway bridge foundations.

UMASS LOWELL

Design & Construction Practices - Questionnaire

Subsurface Conditions for Shallow Foundations

Out of all constructed PIERS, 17% were supported by shallow foundations

Rock 56.3%	IGM 16.3% (cemented soils/ weathered rock)	Frictional Soil 23.9% (sand/gravel)	Cohesive Soil 3.4% (clay/silt)

Cohesive Soil breakdown (%): Alabama-3, Arizona-10, Georgia-5, Idaho-10, Illinois-2, Indiana-20, Michigan-50, Massachusetts-4, Nevada-5, Washington-10

Of those built on cohesive soils, 68% were built without ground improvement measures (geosynthetic, wick drains, etc.)

Out of all constructed ABUTMENTS, 19% were supported solely by shallow foundations:

Rock 55.3%	IGM 17.3% (cemented soils/ weathered rock)	Frictional Soil 24.4% sand/gravel	Cohesive Soil 3.0% clay/silt

Cohesive Soil breakdown (%): Arizona-5, Georgia-5, Idaho-10, Illinois-10, Michigan-25, Massachusetts-2, Nevada-10, Oregon-1, Vermont-10, Washington-10, CA (Alberta)-10

Of those abutments built on cohesive soils, 50 % were built without ground improvement measures (geosynthetic, wick drains, etc.) Georgia-100, Idaho-100, Michigan-100, Massachusetts-80, Nevada-90, Vermont-50, Washington-5, CA (Alberta)-25

28% have integral bridge abutments supported on shallow foundations (about 25% of all integral abutments), while 68% do not use shallow foundations at all.

Design & Construction Practices - Questionnaire

Subsurface Conditions for Shallow Foundations

In summary, 55.8% of the shallow foundations are built on rock (average of piers and abutments) with additional 16.8% on IGM, hence 72.6% of the foundations are build on rock or cemented soils and only 27.4% are built on soils of which 24.2% on granular soils and 3.2% on clay or silt. A further breakdown is presented in Table 1 of Appendix A in the Interim Report.

For example, Michigan indicated that 50% of its shallow foundations at the piers' location are built on fine grained soils, however, Michigan is using only 5% of its pier foundations on shallow foundations; hence, only 2.5% of the pier foundations are built on clay or silt. Examining all the states this way suggests that the leading state to build bridge foundations on clay is WA (6%) followed by VT (5%), ID (4%), and MI and NV (3.75%) each. Further examination of these facts (in a telephone interview) revealed that WA's use of foundations on silt and clay refers to highly glacial densified soils with SPT N values exceeding 30 for silts and between 40 to 100 for the clays.

Design & Construction Practices - Questionnaire

Subsurface Conditions for Shallow Foundations

Twenty-eight states (out of 39) do not build shallow foundations for bridges on cohesive soils at all; hence only 0.8% of all bridge shallow foundations are built on clay or silt including WA, in comparison to 16.9% on rock, 5.4% on IGM and 12.2% on frictional soils. The survey also suggests that only about 60% of the foundations on clay were built without ground improvement measures, hence only about 0.48% of the bridges were actually built on shallow foundations on cohesive soils, practically a marginal number considering the state of these soils as described by WA DOT.

Note – these numbers do not include the construction of embankments and the B.C. evaluation of embankments and do not consider the issue of ϕ - c materials.

Design & Construction Practices - Questionnaire

Foundations on Rock - Implementation

- **About 90% of the states obtain rock cores, evaluate RQD and conduct uniaxial (unconfined) compressive strength tests.**
- **About 19% of the states use presumptive values alone, 22% use engineering analyses alone and 59% use both when evaluating B.C.**
- **53% use AASHTO's presumptive values. Other states use or consult the Canadian Foundation Engineering Manual, NY Building Code, NAVFAC, or based their capacity values on local experience (SD, WI, OR, KS, IA, AK).**
- **70% of the responding states would like to see a specific analytical method presented for the evaluation of B.C. of foundations on rock. 25% use Kulhawy and Goodman (1987) analytical method and 33% use Carter and Kulhawy semi-empirical design method. Others use: Kulhawy and Goodman (1980) Hoek-Brown, Hoek and Marinos. Two states commented about using GSI (Geotechnical Strength Index) instead of RMR (Rock Mass Rating).**
- **60% evaluate failure by sliding for footings on rock. Seven states do not evaluate sliding because of a requirement to "wedge" the foundation into the rock**

Design & Construction Practices - Questionnaire

Foundations on Rock - Implementation

- **70%** of the states do not analyze lateral displacement as they use limiting measures (key way, dowling, etc.) as described above. NY specifies geologic inspection during construction to ensure rock quality and key way or dowelling is ordered if necessary.
- **75%** of the responding states limit the eccentricity of footings on rock. Most of the states follow AASHTO recommendations for $e/B \leq 3/8$, some use $e/B \leq 1/4$ based on the FHWA "Soils and Foundations Manual" that also meets the AASHTO standards specification. WY, SD, and Alberta use $e/B \leq 1/6$ with Alberta specifying that either eccentricity is maintained within limits or an effective foundation size is used in which the dimensions are reduced by twice the eccentricity (e.g. $B' = B - 2e$).
- **70%** of the states do not analyze settlement of footings on rock as it is not being seen as an issue of importance and the settlement is limited to 0.5in. 28% use AASHTO procedures for broken/jointed rock with NV also using Kulhawy (1987) and the Army EM 110-1-2908.

Note – questionnaire did not address differences between competent/hard rock and soft rock/IGMs

Design & Construction Practices - Questionnaire

Foundations on Soil - Implementation

- **All states follow either AASHTO's LRFD or ASD guidelines, only a small number of responders use presumptive values. 58% use the theoretical general B.C. equation.**
- **53% of the responders find it reasonable to omit the load inclination factors and 63% limit the eccentricity of the footing mostly with e/B ≤ 1⁄6 to ¼ (standard specifications e/B = 1⁄6, LRFD specifications e/B = ¼). MA responded that load inclination factors must be used in the final design of the footing. PA commented that when inclination factors were considered together with factored loads, it resulted in an increased footing size; hence, unfactored loads are used.**
- **45% do not decrease the soil's strength parameters considering punching shear, while 23% do so. Seven states commented that punching shear is not a viable option as foundations are not built in loose soil conditions or alternatively settlement criteria prevails especially under such conditions.**
- **58% use the AASHTO procedures presented for footings on a slope. NV, ID and MI commented that the charts are not clear and needs to be improved. WA, and NC commented on the use of Meyerhoff's method, also presented by the Navy Design Manual (NAVFAC), essentially identical to the AASHTO presentation. OR commented that the provided foundations on slope analysis results with a reasonable approach (somehow conservative) while PA commented that experience shows that sometimes this analysis results with a drastically larger footing.**

Design & Construction Practices - Questionnaire

Foundations on Soil - Implementation

- **30% of the responding states do not use the AASHTO procedures for footings on a layered soil, while 38% of the responders do use these procedures. Eighteen states commented about the procedures; ID, MI, VT and WI commented that they calculate the B.C. for the layer with the lower strength. IA and OR commented that under such conditions alternative foundation solutions are examined.**
- **Only 28% (with 40% responding with No) of the responders use the semi-empirical procedures described in section 10.6.3.1.3 for evaluation of bearing capacity. The majority of the states that commented about the procedure expressed the opinion that the method is used for a rough evaluation only as an initial estimation and/or in comparison to other methods. Oregon commented that the SPT method usually yields higher capacity and settlement controls the design.**
- **Nineteen states responded when asked for comments about the currently existing resistance factors being all about the same value. Some states stated that they have not enough experience with LRFD to judge the resistance factors values. NC and NH suggested combining all resistance factors to be 0.45, while OR, PA, VT, and WA commented that the resistance factors' are in line with the factor of safety range (2.5 to 3.0) used in the ASD methodology and hence result with similar design as that obtained using ASD.**
- **70% evaluate failure by sliding with about half (33%) use the full foundation area and 30% use the effective foundation area.**

Design & Construction Practices - Questionnaire

Foundations on Soil - Implementation

- Only 13% consider passive resistance for the lateral resistance of the shallow foundations and all utilize a limited value due to a limited displacement. Many responding states expressed concern with a long term reliance on a passive resistance. WA commented that it is rarely used to meet sliding criterion of extreme events and MN commented it is used in front of shear keys only.
- Traditionally no safety margin is provided to settlement analysis though it typically controls the size of shallow foundations. When asked about it, 35% answered the issue should not be of concern and 25% answered it should. From those responded, some recognized that it needs to be researched (CT, MI, TN) while others hold the notion that a safety margin on B.C. already addresses the issue (HI, ME, NJ, NC, WA) or that settlement calculations are conservative to begin with (NH, NC).
- Only two states stated that they conduct plate load tests, one of which (CT) referred to tests from over 20 years ago and the other to three recent tests (MA).
- When asked to comment on any related subject, 13 states responded. A major concern expressed by MI was written by a bridge designer referring to the difficulties in using effective width for bearing capacity calculations as it requires iterations for each load case for service and strength. More so, the division of responsibilities between the geotechnical section (providing allowable pressure) and structural section (examining iteratively final design) is a source for problems. The engineer proposes to have allowable contact stresses for service and strength based on gross footing width and eccentricity limited to B/6. (The issue of "allowable" to ULS is not so clear and the engineer was contacted).

DATABASES

- **UML-GTR ShalFound07 Database**

  499 cases built in ACCESS platform, currently being updated to 549 cases. Out of it, 415 cases are suitable for ULS.

- **UML-GTR RockFound07 Capacity Database**

  122 Cases of load tests to failure including 61 rock sockets, 33 shallow foundations on rock surface, 28 shallow foundations below surface

Assembled Databases

UML-GTR ShalFound07 (predominantly granular soils)
499 test cases

- **Database I** — centric vertical loading
- **Database II** — eccentric and inclined loadings and their combinations

UML-GTR RockFound07 (rocks)
122 test cases

- **Database III** — centric vertical loading

BC Shallow Foundations on Soil - OUTLINE

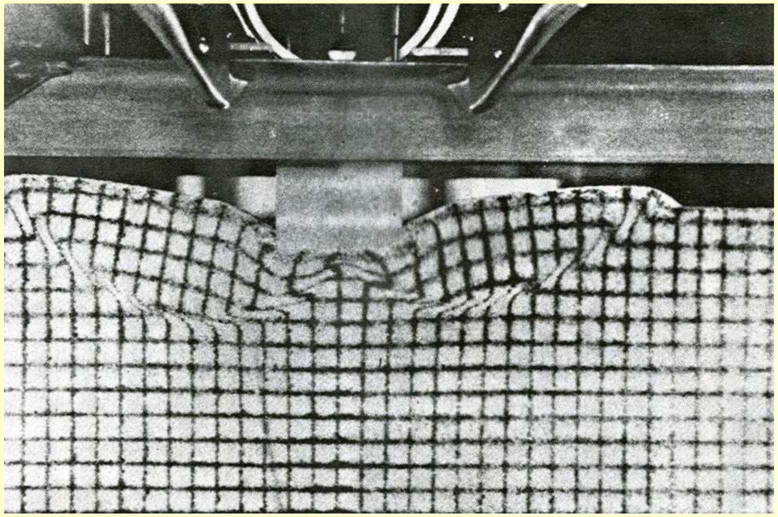
1. **BC of Shallow foundations**
   - **BC Factors**
   - **BC modification Factors**
2. **Determination of ULS from case histories**
   - **ULS and Modes of Failure – Overview**
   - **Modes of Failure**
3. **Failure (Ultimate Load) Criteria**
   - **Minimum slope criteria (Vesić, 1963)**
   - **Limited settlement criterion of 0.1B (Vesić, 1975)**
   - **Log-log plot of load-settlement curve (DeBeer, 1967)**
   - **Two-slope criterion**
   - **Selection of failure criteria (representative values and minimum slope)**
   - **Examples in soil & rock**
4. **Uncertainty Evaluation – BC of Centric Vertically Loaded Footing on Granular Soils**
   - **Database overview**
   - **Calculated BC – missing soil parameters and equations used for BC calculations**
5. **Calibration**
6. **Summary and Conclusions**

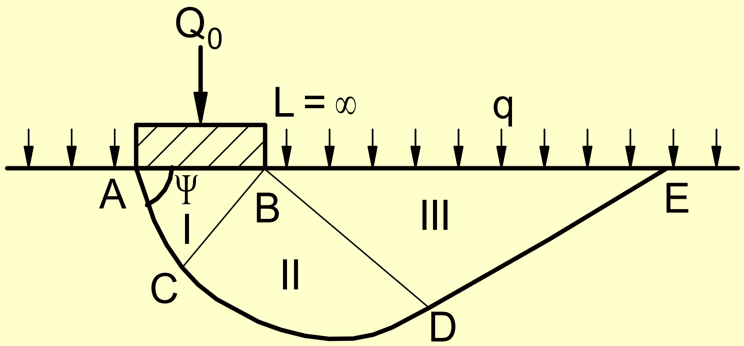
Bearing Capacity of Shallow Foundation

General Formulation

- **Centric vertical loading of a rigid footing**
  - **Buismann (1940) and Terzaghi (1943) adopted solution for metal punching proposed by Prandtl (1920, 1921) and proposed the Ultimate Bearing Capacity**

$$q_u = c \cdot N_c + q \cdot N_q + \tfrac{1}{2} \cdot \gamma \cdot B \cdot N_\gamma$$





Bearing Capacity Factors N_c and N_q

- **These factors have exact solutions and were given by Prandtl (1920) and Reissner (1924) for weightless soils**
- **proposal for N_c is credited to Caquot and Kerisel (1953)**

$$N_c = \left(N_q - 1\right)\cot\phi_f$$

$$N_q = \exp\left(\pi \tan\phi_f\right)\cdot \tan^2\left(\frac{\pi}{4} + \frac{\phi_f}{2}\right)$$

Bearing Capacity Factor N_γ

- **No closed form solution present and proposals from different authors exist**
- **Formulas based on Empirical Relations**

Meyerhof (1963): $N_\gamma = (N_q - 1) \cdot \tan(1.4\phi_f)$

Brinch Hansen (1970): $N_\gamma = 1.5(N_q + 1) \cdot \tan \phi_f$

Muhs (1971) and Eurocode 7 (2005): $N_\gamma = 2(N_q - 1)\tan\phi_f$

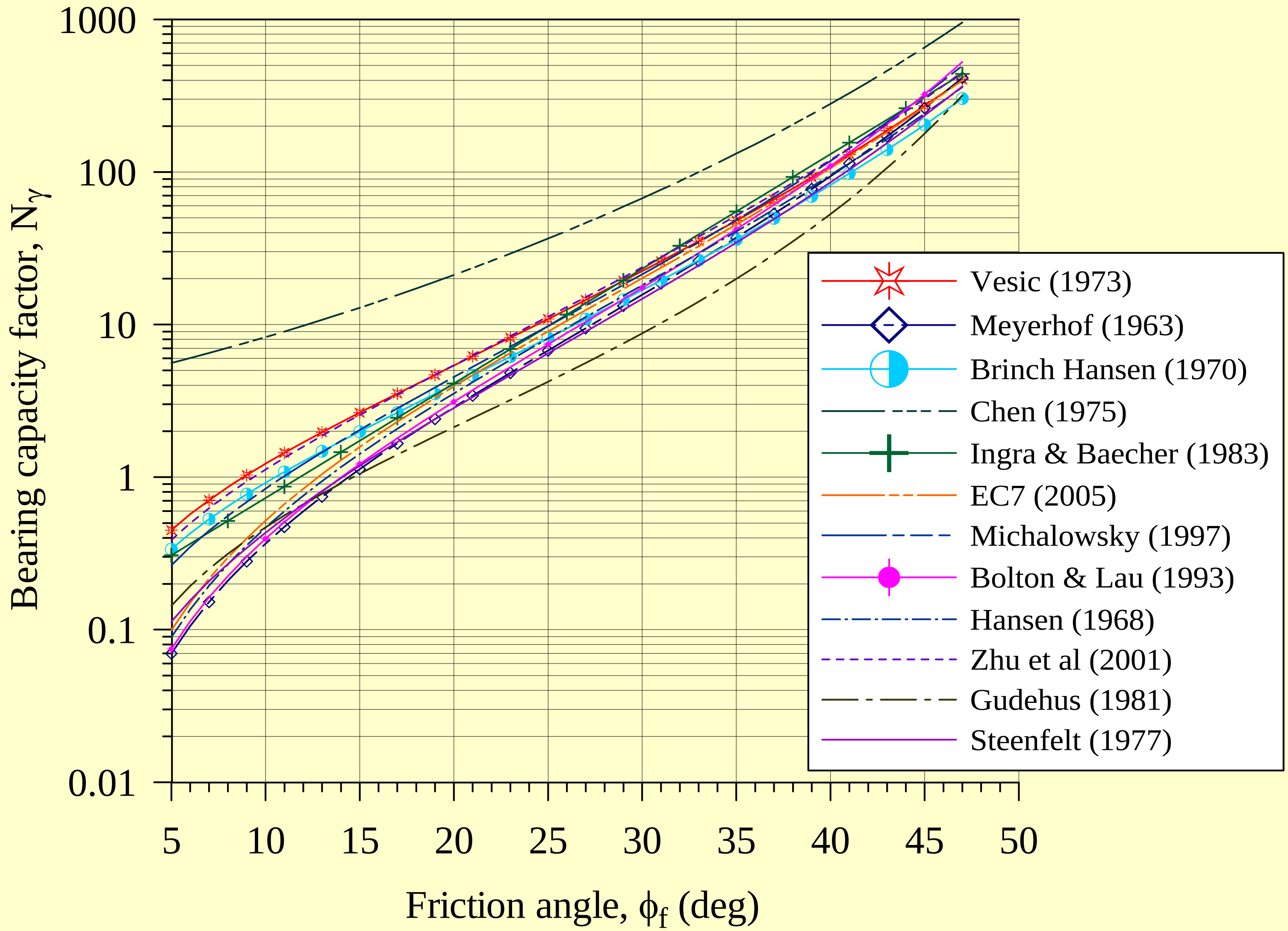
Ingra and Baecher (1983) for square footings: $N_\gamma = \exp(-2.046 + 0.173 \cdot \phi_f)$

- **Formulas based on Analytical Derivation**

Vesic (1973): $N_\gamma = 2(N_q + 1) \cdot \tan \phi_f$

For a complete list, refer to NCHRP Report 651

Different Proposed Nγ Factors



B.C. factor N_γ vs. ϕ based on empirical, analytical and numerical derivations

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General Bearing Capacity Equation

Based on modifications by Meyerhoff (1953, 1963), Brinch Hansen (1961, 1970) and Vesić (1973, 1975):

$$q_u = c \cdot N_c \cdot s_c \cdot d_c \cdot i_c + q \cdot N_q \cdot s_q \cdot d_q \cdot i_q + \tfrac{1}{2} \cdot \gamma \cdot B' \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma \cdot i_\gamma$$

where

s, d, and i are modification factors for footing shape, footing embedment depth and load inclination, respectively,

effective width B′ = B - 2e

e = load eccentricity

Eccentricity

The effect of eccentric loading on the bearing capacity is usually accounted for via Meyerhof's (1953) effective area consideration. The bearing capacity is calculated for the footings' effective dimensions given by:

$$L' = L - 2 \cdot e_L \qquad \text{with } e_B = M_L/V \text{ and } e_L = M_B/V \qquad (35)$$
$$B' = B - 2 \cdot e_B$$

where M, M_B and M_L = the moments loading in L and B directions, respectively
V = the total vertical load
e_L and e_B = load eccentricities along footing length L and footing width B, respectively.

Meyerhof (1953): $$\frac{q_u}{q_{u,centric}} = \left(1 - 2\frac{e}{B}\right)^2 \qquad (36)$$

Giraudet (1965): $$\frac{q_u}{q_{u,centric}} = \exp\left(-12\left(\frac{e}{B}\right)^2\right) \qquad (37)$$

Ticof (1977): $$\frac{q_u}{q_{u,centric}} = \left(1 - 1.9\frac{e}{B}\right)^2 \qquad (38)$$

Eccentricity

Bowles (1996): $$\frac{q_u}{q_{u,centric}} = 1 - \sqrt{\frac{e}{B}} \quad \text{for } 0 < \frac{e}{B} < 0.3 \qquad (39)$$

Paolucci and Pecker (1997): $$\frac{q_u}{q_{u,centric}} = \left(1 - \frac{e}{0.5B}\right)^{1.8} \quad \text{for } \frac{e}{B} < 0.3 \qquad (40)$$

Ingra and Baecher (1983): $$\frac{q_u}{q_{u,centric}} = 1 - 3.5\left(\frac{e}{B}\right) + 3.03\left(\frac{e}{B}\right)^2 \qquad (41)$$

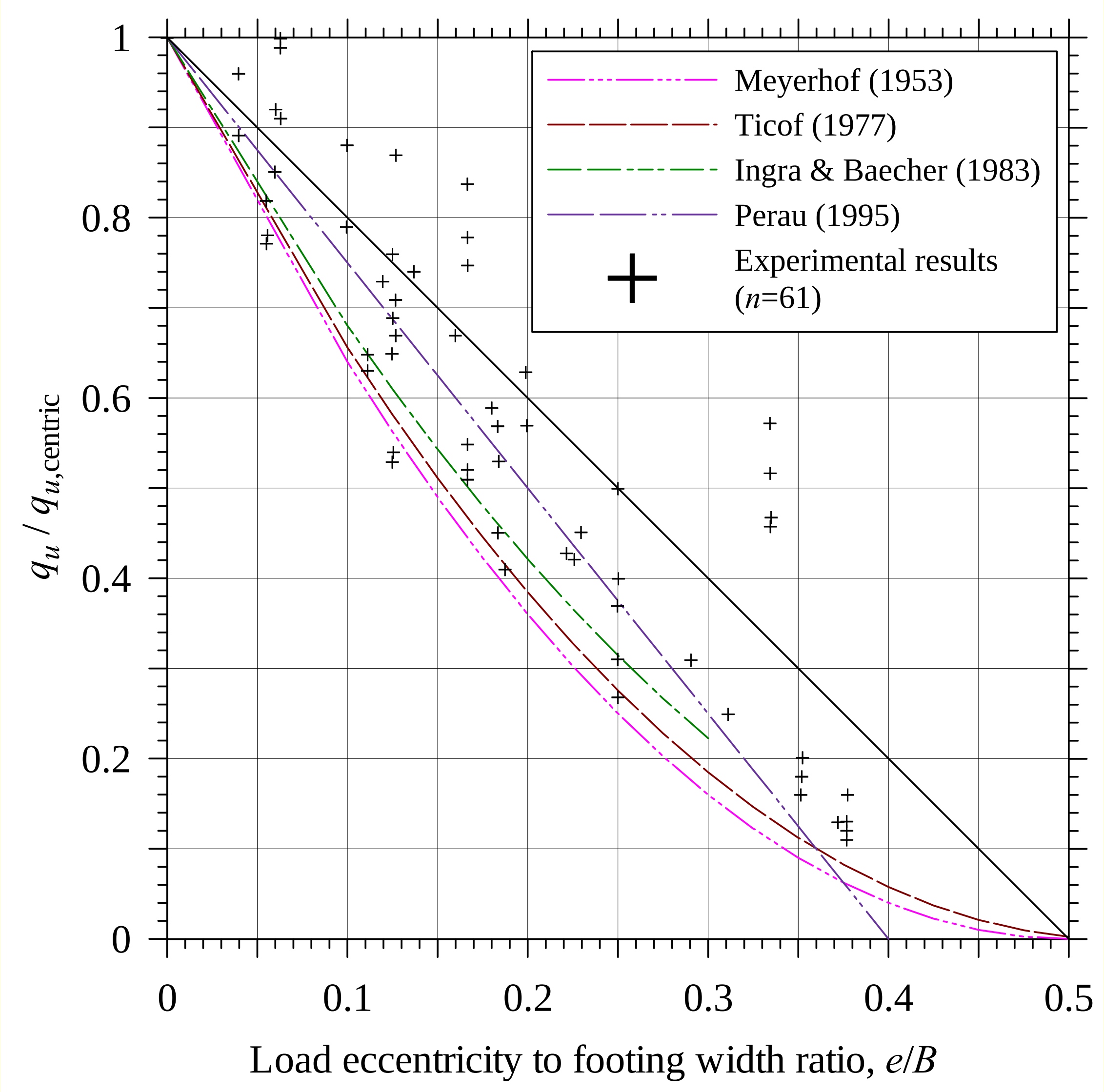
Gottardi and Butterfield (1993): $$\frac{q_u}{q_{u,centric}} = 1 - \frac{e}{0.36B} \qquad (42)$$

Perau (1995, 1997): $$\frac{q_u}{q_{u,centric}} = 1 - 2.5\frac{e}{B} \qquad (43)$$

Eccentricity: Reduction factors for Foundations Under Vertical-Eccentric Loading

Test were carried on footings with different length to width ratios.

It can be seen that the Meyerhof's proposal is closest to the lower boundary of the test results.



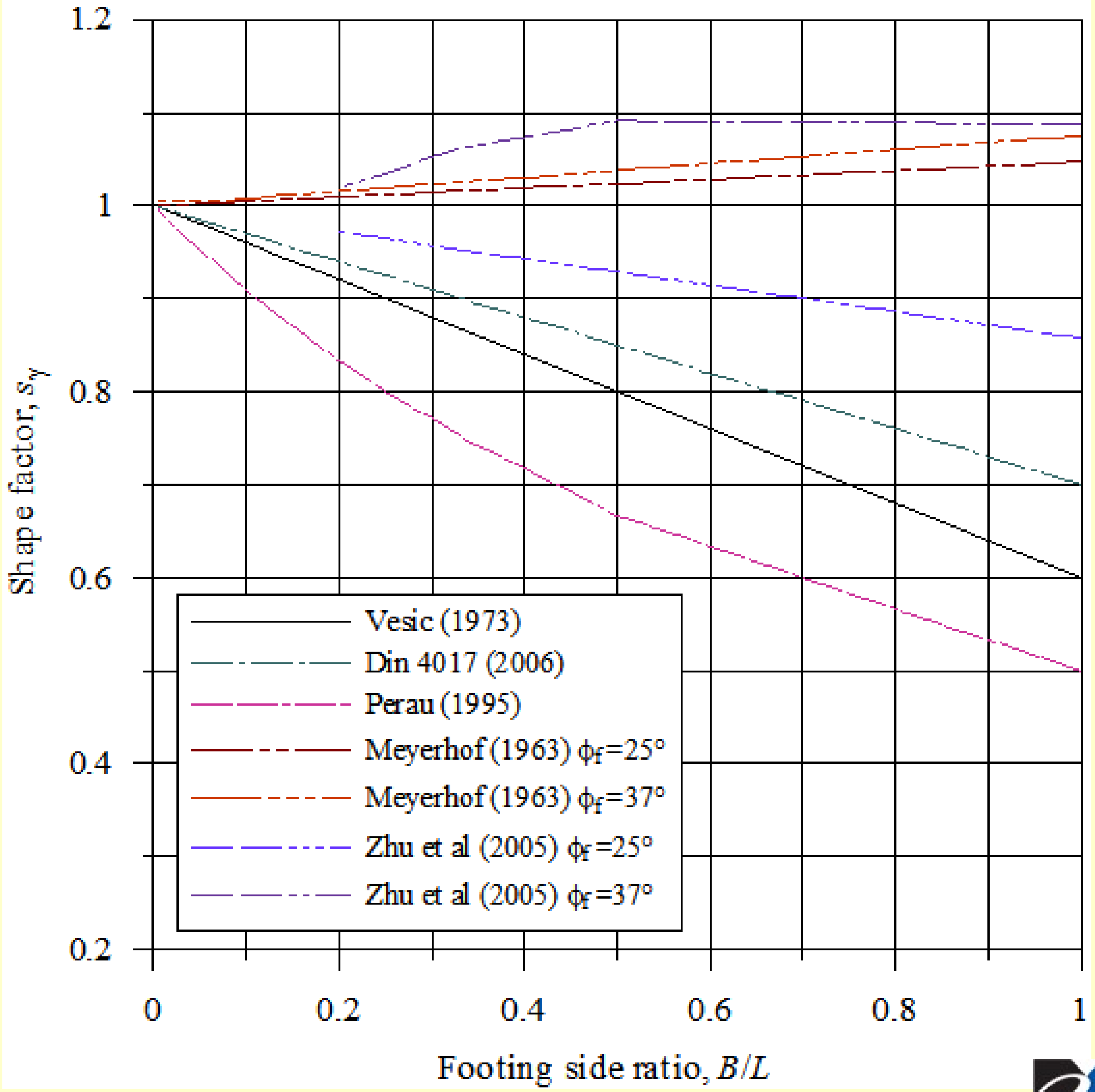
Shape Factors

Table 4 Shape factors proposed by different authors

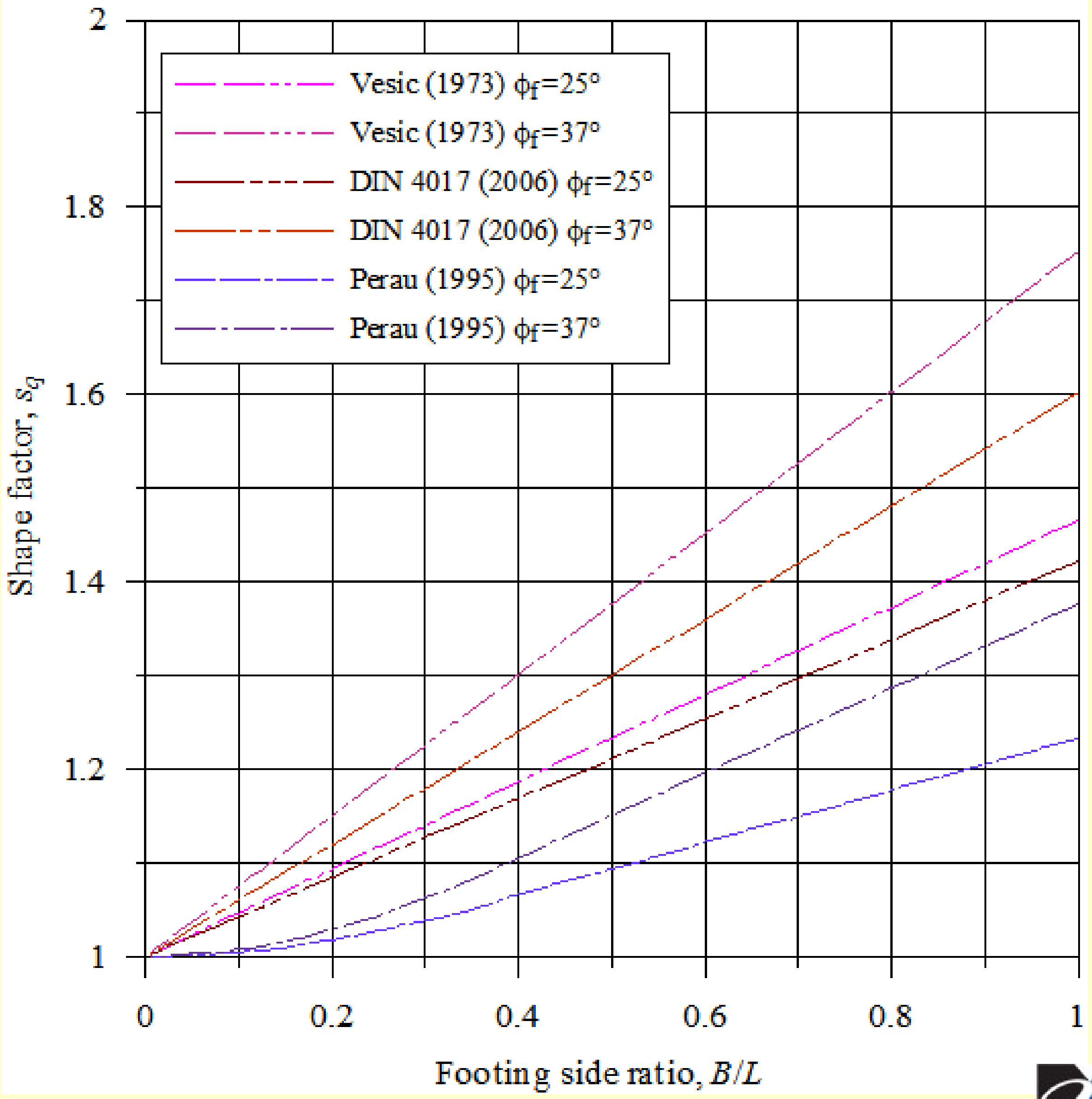
Reference	Footing base shape	s_c	s_q	s_γ
De Beer (1961) as modified by Vesic (1973)	Rectangle	$1+\frac{B'}{L'}\frac{N_q}{N_c}$	$1+\frac{B'}{L'}\cdot\tan\phi_f$	$1-0.4\frac{B'}{L'}$
	Circle and Square	$1+\frac{N_q}{N_c}$	$1+\tan\phi_f$	0.6
EC 7 (2005) and DIN 4017 (2006)	Rectangle	$\frac{(s_q\cdot N_q-1)}{N_q-1}$	$1+\frac{B'}{L'}\cdot\sin\phi_f$	$1-0.3\frac{B'}{L'}$
	Circle and Square	$\frac{(s_q\cdot N_q-1)}{N_q-1}$	$1+\sin\phi_f$	0.7
Meyerhof (1963)	Rectangle	$1+0.1\frac{B'}{L'}\cdot K_p$	$=1$; for $\phi_f=0$ $=1+0.1K_p(B'/L')$; for $\phi_f>10^\circ$	$1+0.1\frac{B'}{L'}\cdot K_p$; $K_p=\tan^2\left(45^\circ+\frac{\phi_f}{2}\right)$
Perau (1995, 1997)	Rectangle	–	$1+1.6\tan\phi_f\cdot\frac{B'/L'}{1+\left(\frac{B'}{L'}\right)^2}$	$\frac{1}{1+\frac{B'}{L'}}$
Zhu and Michalowski (2005)	Rectangle	–	–	$1+(0.6\sin^2\phi_f-0.25)B'/L'$ for $\phi_f\le 30^\circ$; $1+(1.3\sin^2\phi_f-0.5)(L'/B')^{1.5}\cdot\exp(-L'/B')$ for $\phi_f>30^\circ$

Shape Factor: s_γ

The value of s_γ is within the range of 1 ± 0.05 for $L/B \geq 6.7$



Shape Factor: s_q

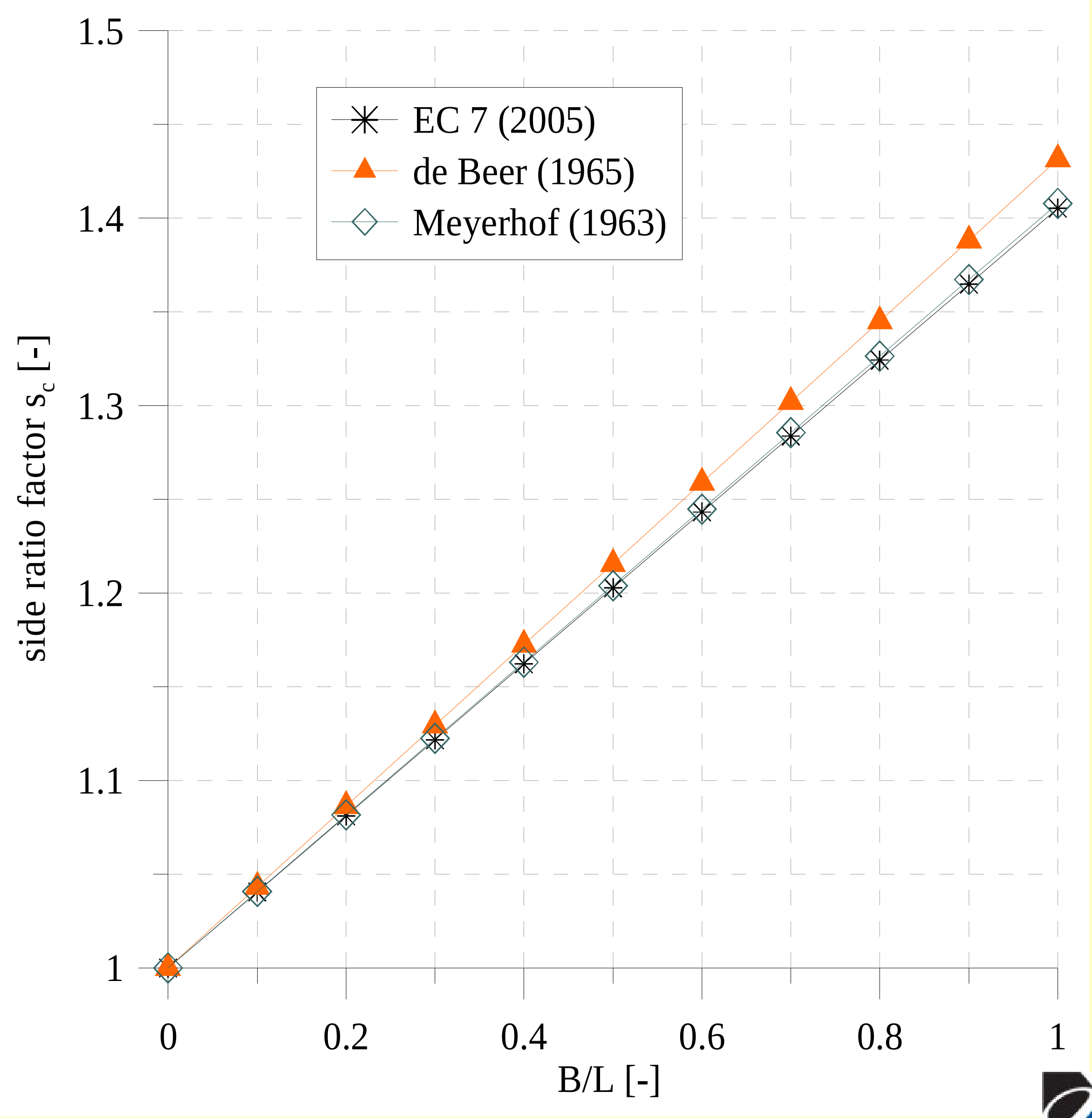


The value of s_q is within the range of 1 ± 0.05 for $L/B \geq 10.0$

Shape Factor: s_c

For soil with ϕ_f=20° and c′=5kPa (0.1ksf)

The value of s_c is within the range of 1±0.05 for L/B ≥ 10.0

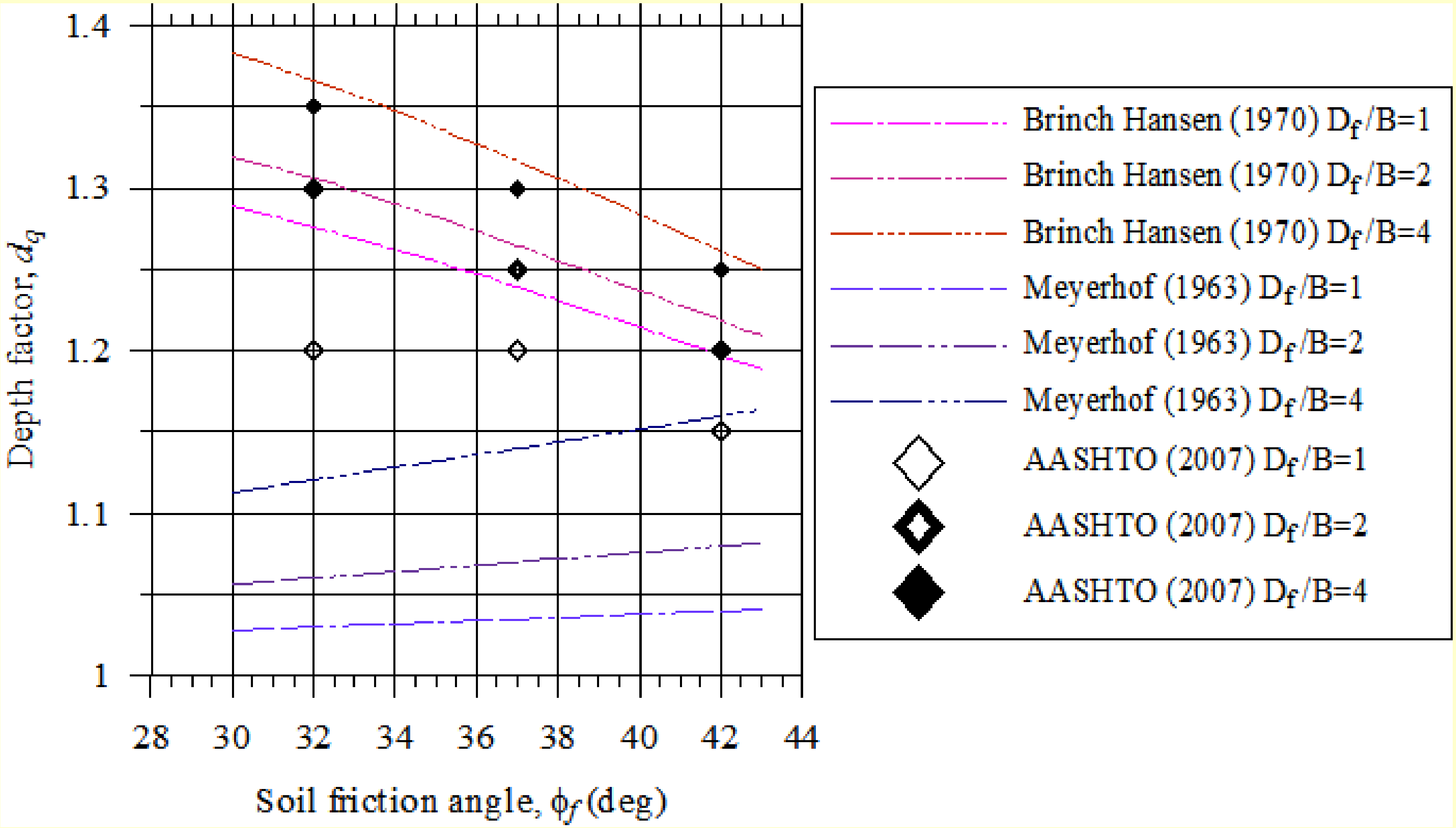


Depth Factors

Table 5 Depth factors proposed by different authors

Author	d_c	d_q	d_γ
Meyerhof (1963)	$d_c = 1 + 0.2\sqrt{K_p} \cdot \frac{D_f}{B'}$	$d_q = 1 + 0.1\sqrt{K_p} \cdot \frac{D_f}{B'}$ for $\phi_f > 10$ $= 1$ for $\phi_f = 0$	$d_\gamma = d_q$
Brinch Hansen (1970) and Vesic (1973)	$d_c = d_q - \frac{1 - d_q}{N_C \cdot \tan\phi_f}$ $= d_q - \frac{1 - d_q}{N_q - 1}$	$D_f / B' \leq 1$: $d_q = 1 + 2\tan\phi_f \cdot (1 - \sin\phi_f)^2 \cdot (D_f / B')$ $D_f / B' > 1$: $d_q = 1 + 2\tan\phi_f \cdot (1 - \sin\phi_f)^2 \, arc\tan(D_f / B')$	1
where $K_p = \tan^2(45° + \phi_f / 2)$			

Depth Factor: d_q



Load Inclination Factors

An inclination in the applied load always results in a reduced bearing capacity, often of a considerable magnitude (Brinch Hansen, 1970). Meyerhof (1953) suggested that the vertical component of the bearing capacity under a load inclined at an angle α to the vertical, is obtained using the following inclination factors.

$$i_c = i_q = \left(1 - \alpha / 90°\right)^2 \quad (44)$$

$$i_\gamma = (1 - \alpha / \phi_f)^2 \quad (45)$$

These expressions were modified by Meyerhof and Koumoto (1987), and presented for the cases of footings on the surface of sand, when embedment ratio (D_f / B) is unity, and for footings on the clay surface as given below. Assuming that a footing with a perfectly rough base on the sand surface starts to slide when the load inclination angle to the vertical is approximately equal to the soil's friction angle, the following expression was proposed:

$$i_\gamma = \cos\alpha \left(1 - \frac{\sin\alpha}{\sin\phi_f}\right) \qquad \text{for } D_f / B' = 0, \quad c = 0 \quad (46)$$

Load Inclination Factors

For a particular case of footings with embedment ratio equal to 1 in a soil with friction angle greater than 30°, the inclination factor was expressed as:

$$i_\gamma = \cos\alpha\left(1-\sin\alpha\right) \qquad \text{for } \phi_f > 30°,\ D_f / B' = 1,\ c = 0 \qquad (47)$$

For footings on the surface of clay:

$$\begin{aligned} i_c &= \cos\alpha\left(1-\sin\alpha\right) && \text{for } c_a = 0 \\ &= \cos\alpha\left(1-0.81\sin\alpha\right) && \text{for } c_a = c_n = \text{undrained shear strength of the clay} \end{aligned} \qquad (48)$$

where c_a = adhesion between the clay and the base of the footing

Muhs and Weiss (1969) suggested, based on DEGEBO (Deutsche Forschungsgesellschaft für Bodenmechanik) tests with large scale models of shallow footings on sands, that there is a distinct difference in the load inclination effects when the inclination is in the direction of the longer side *L* and when in the direction of the shorter side *B*. Thus, the direction of load inclination as well as the ratio *B*/*L* affect on the inclination factor. Brinch Hansen (1970) incorporated the inclination effects as:

$$i_q = \left(1 - \frac{0.5H}{\left(V + A'c\cot\phi_f\right)}\right)^5 \qquad (49)$$

$$i_\gamma = \left(1 - \frac{0.7H}{\left(V + A'c\cot\phi_f\right)}\right)^5 \qquad (50)$$

Load Inclination Factors

Vesic (1975) proposed the factors in the following forms:

$$i_q = \left(1 - \frac{H}{\left(V + A'c\cot\phi_f\right)}\right)^n \quad (51)$$

$$i_\gamma = \left(1 - \frac{H}{\left(V + A'c\cot\phi_f\right)}\right)^{n+1} \quad (52)$$

$$n = \left[\frac{\left(2 + L'/B'\right)}{\left(1 + L'/B'\right)}\right]\cos^2\theta + \left[\frac{\left(2 + B'/L'\right)}{\left(1 + B'/L'\right)}\right]\sin^2\theta \quad (53)$$

where *H* and *V* are the horizontal and vertical components of the applied inclined load *P* (Figure 17), θ is the projected direction of the load in the plane of the footing, measured from the side of length *L* in degrees; *L′* and *B′* as defined in Equation 35, *A′* is the effective area of the footing, and *c* is soil cohesion.

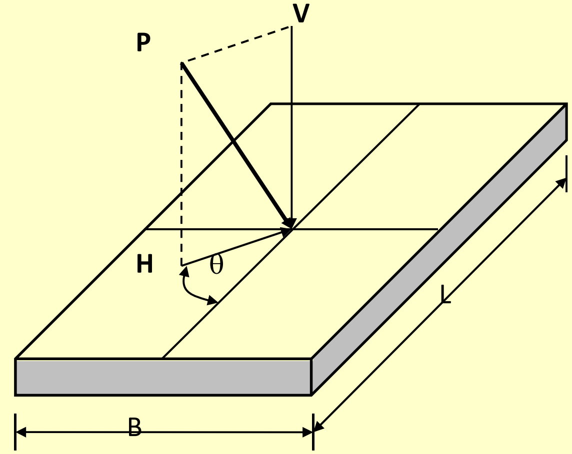
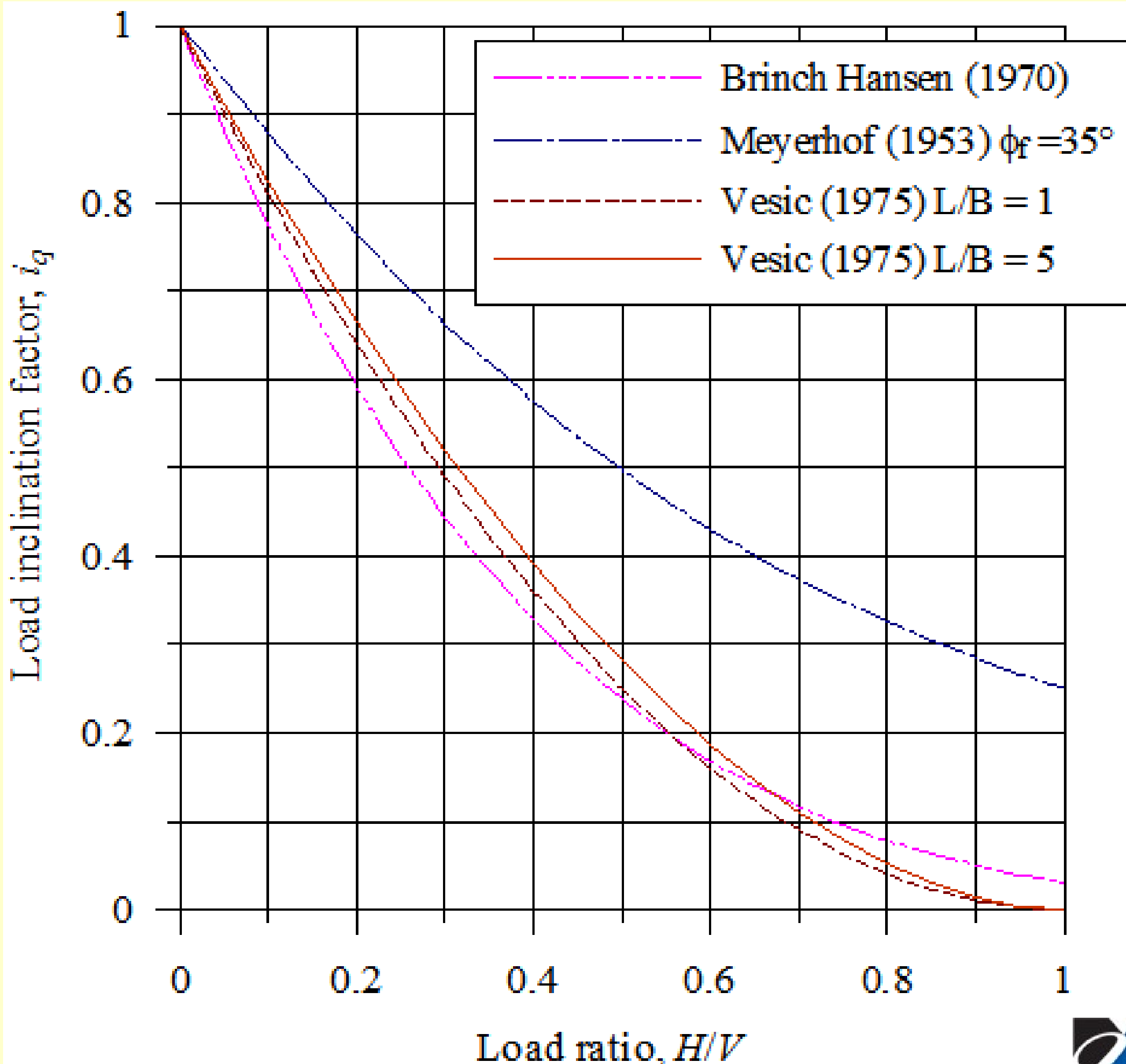


Figure 17 Inclined load without eccentricity, and the projected direction θ

Load Inclination Factor: i_q

Plots valid for horizontal component of load normal to the footing length

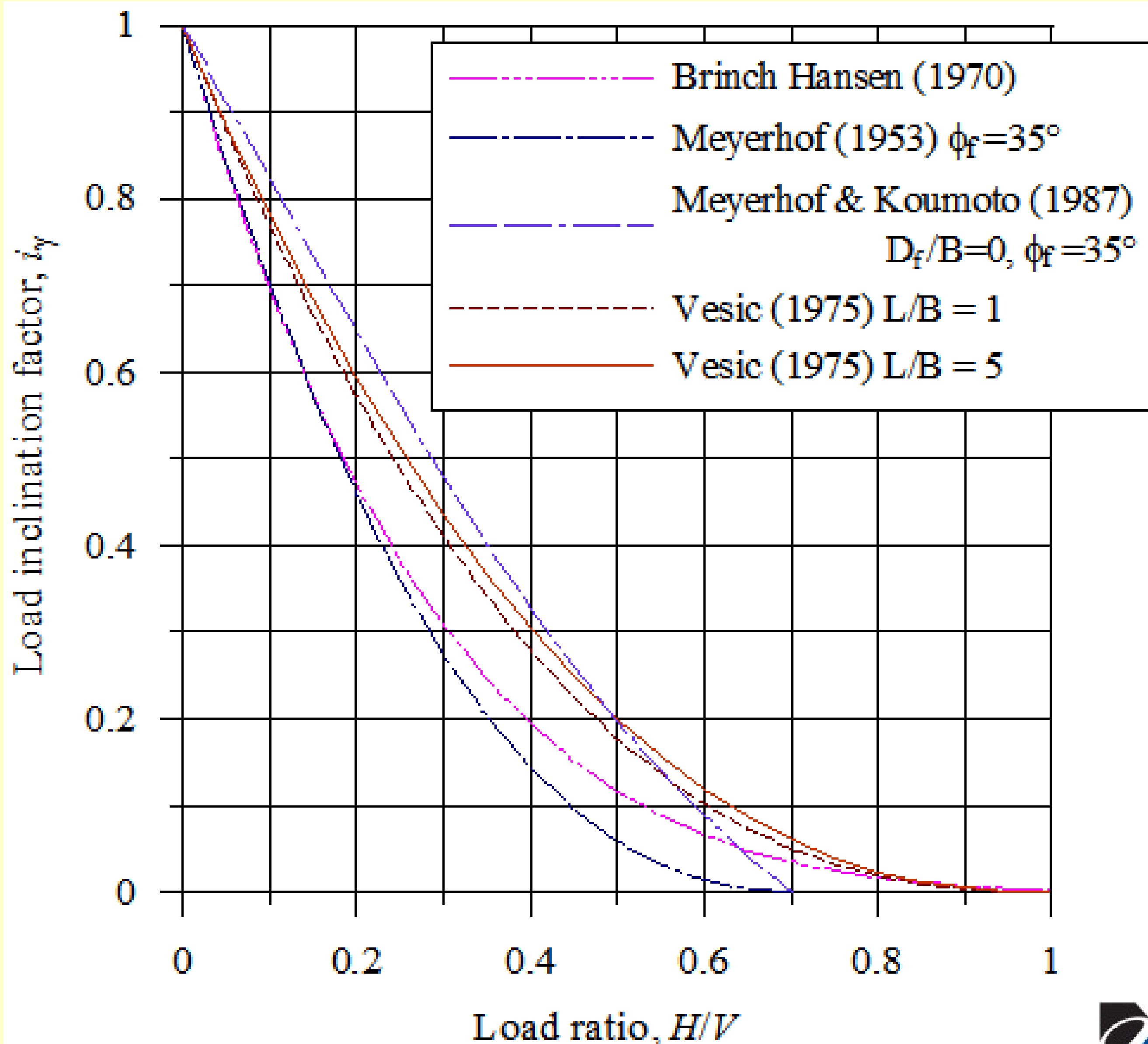
c=0, ϕ_f=35° , and D_f/B=0



Load Inclination Factor: i_γ

Plots valid for horizontal component of load normal to the footing length

c=0, ϕ_f=35° , and D_f/B=0



Load Inclination Factors

The inclination factor i_c results from Caquot's theorem of corresponding stress states (De Beer and Ladanyi 1961 and Vesić 1970 as cited by Vesić 1975) are:

$$i_c = i_q - \frac{1-i_q}{N_c \tan\phi_f} = i_q - \frac{1-i_q}{N_q - 1} \quad \text{for } \phi_f > 0 \tag{54a}$$

$$i_c = 1 - \frac{nH}{A'c\,N_c} \quad \text{for } \phi_f = 0 \tag{54b}$$

where i_q is given by Equation 51.

Reduction coefficients for the case of a load inclination related to the case of a centrically and vertically loaded footings can be found in the references of Figure 20. These expressions were determined based on model foundation test results on sand without embedment; and as such, are valid for the case of D_f = 0, c = 0.

Ticof (1977) $$\frac{q_u}{q_{u,centric}} = \left(1 - 1.36\frac{H}{V}\right)^2 \tag{55}$$

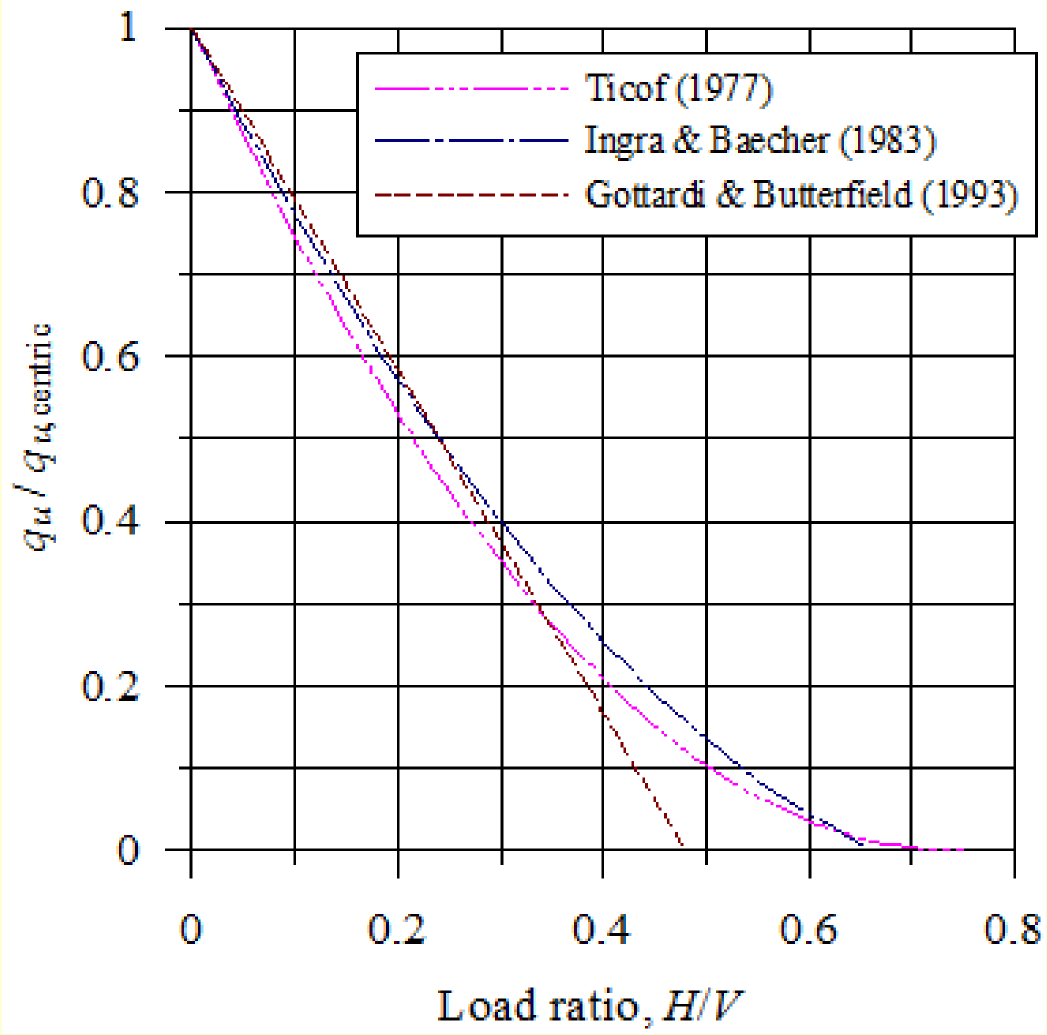
Ingra and Baecher (1983) $$\frac{q_u}{q_{u,centric}} = 1 - 2.41\left(\frac{H}{V}\right) + 1.36\left(\frac{H}{V}\right)^2 \tag{56}$$

Gottardi and Butterfield (1993) $$\frac{q_u}{q_{u,centric}} = 1 - \frac{H}{0.48 \cdot V} \tag{57}$$

Reduction Factor for Load Inclination

Effects of load inclination on ultimate bearing capacity

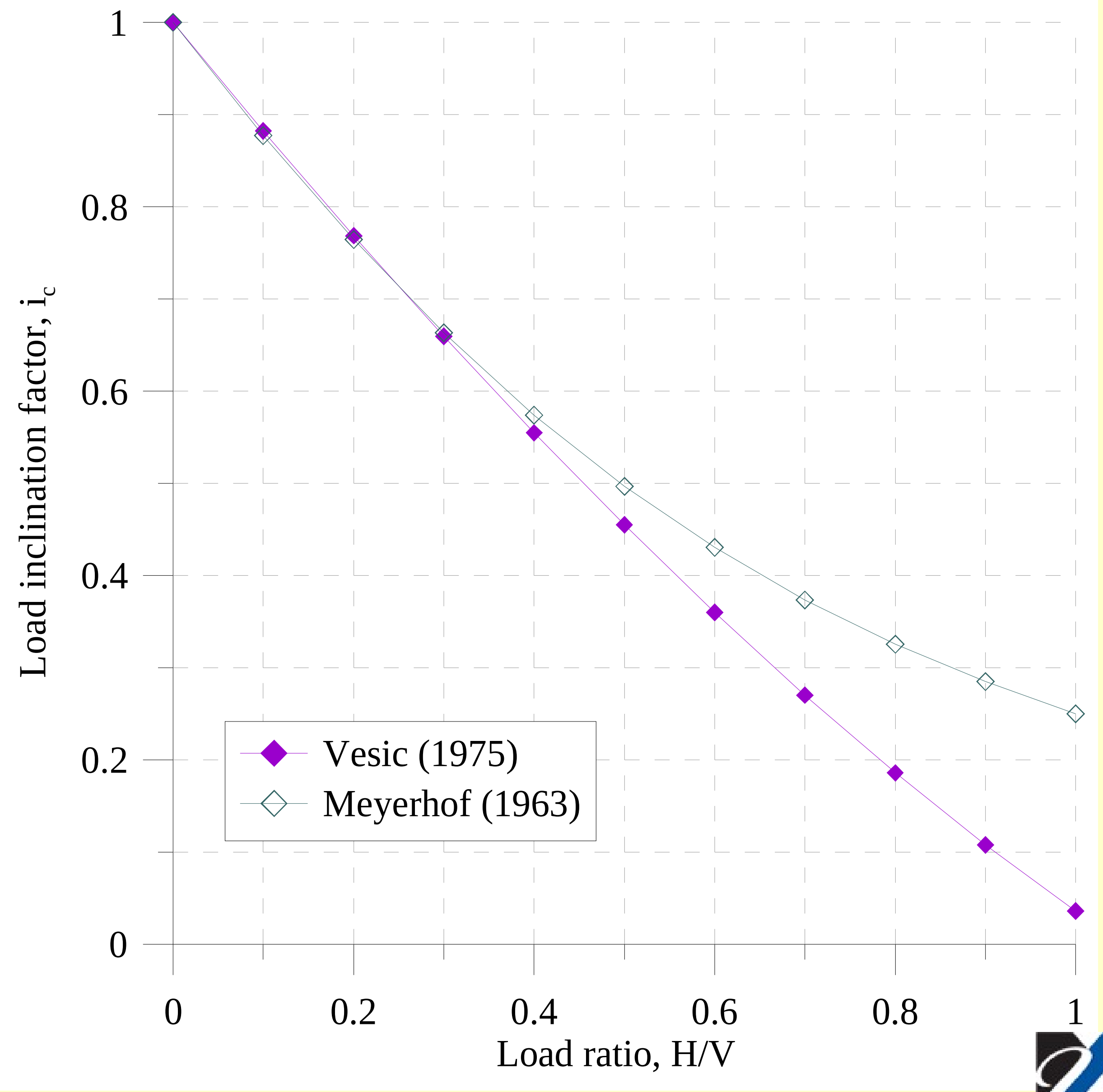
- Based on model tests on sands: D_f=0, c′=0 inclined vs. vertical-centric



Load Inclination Factor: i_c

Plotted for square footings, with base area of $1m^2$ ($10.75ft^2$) on soil with $\phi_f=20°$ and $c'=5kPa$ (0.1ksf)

($V=A\times c'\times \cot\phi_f$)



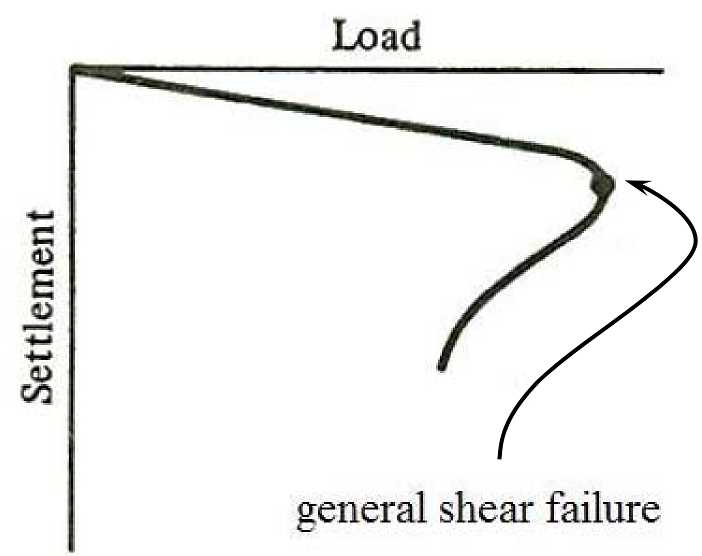
Determination of ULS from Case Histories

- **Ultimate Limit State (ULS) consists of:**
  - **Exceeding load carrying capacity of the ground supporting the foundation**
  - **Sliding, uplift and/or overturning**
- **Three principle modes of shear failure under foundation:**
  - **General shear failure**
  - **Local shear failure**
  - **Punching shear failure**

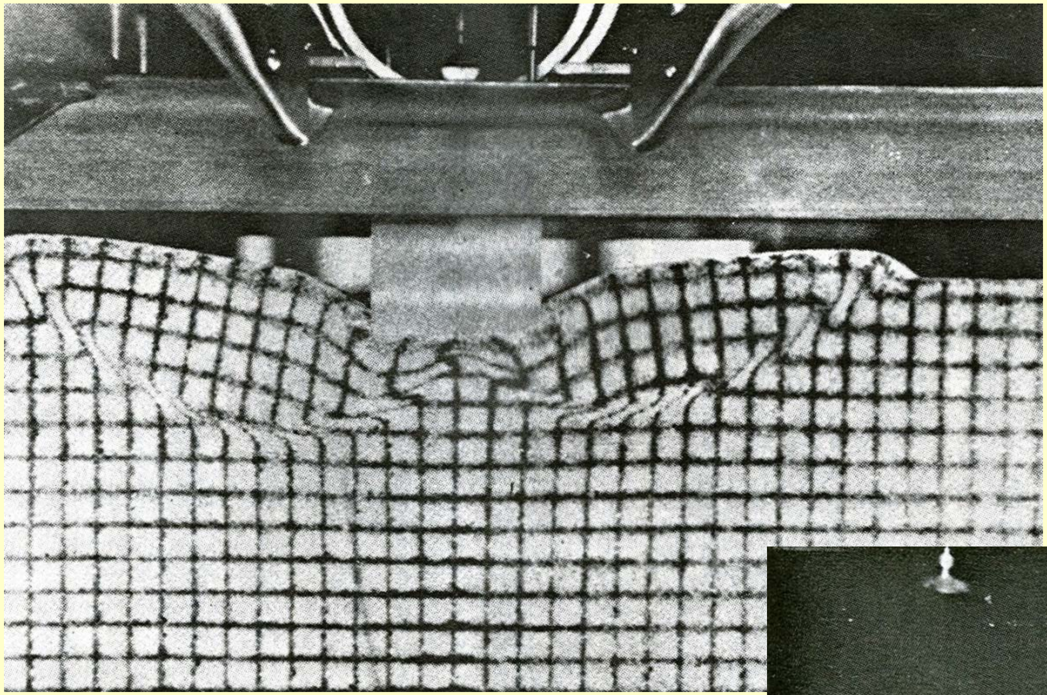
General Shear Failure

- **Characterized by well-defined failure pattern of a continuous slip surface**
- **Load-displacement curve shows a prominent peak**

(Vesic, 1975)

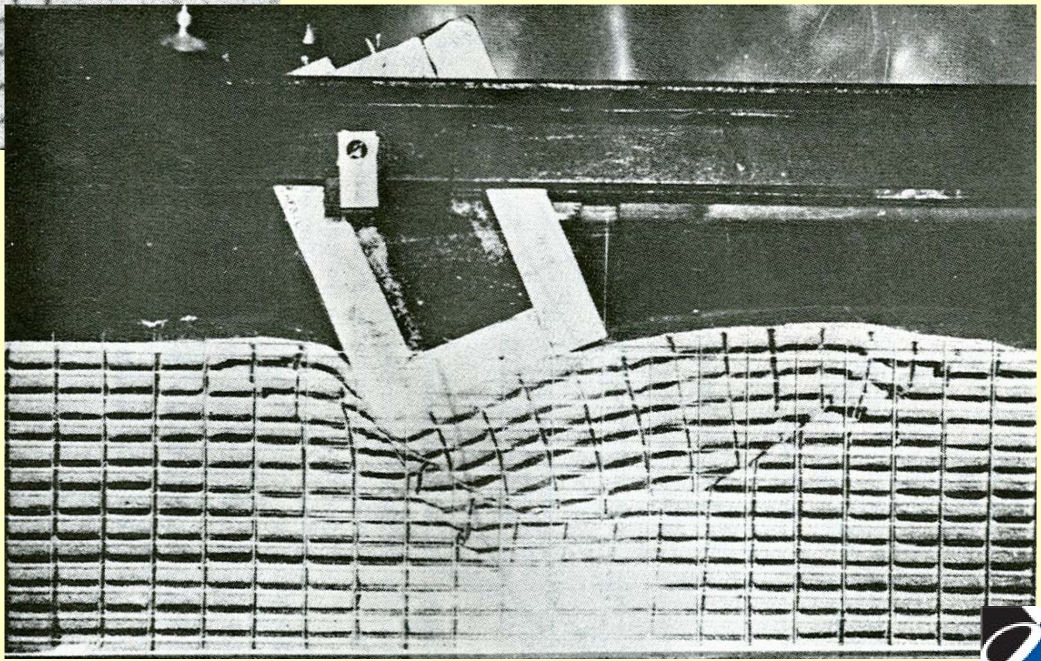


General Shear Failure



Load test of a 3inch footing under centric vertical loading

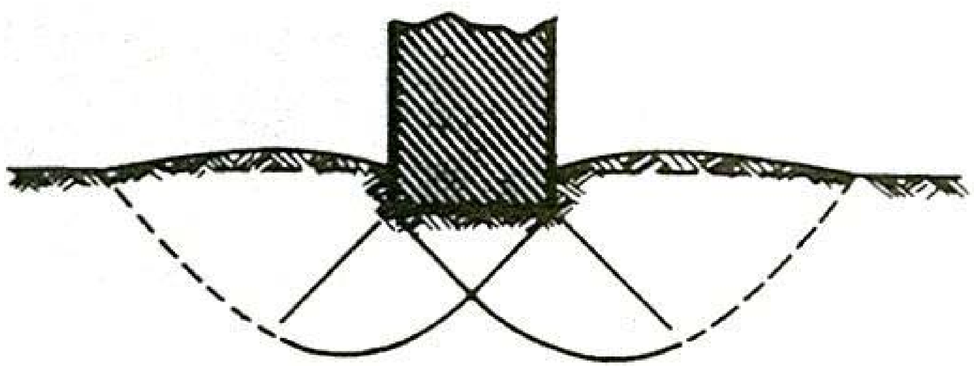
Slip surfaces developed under and on the sides of the footing developed after general shear failure (Selig and McKee, 1961)

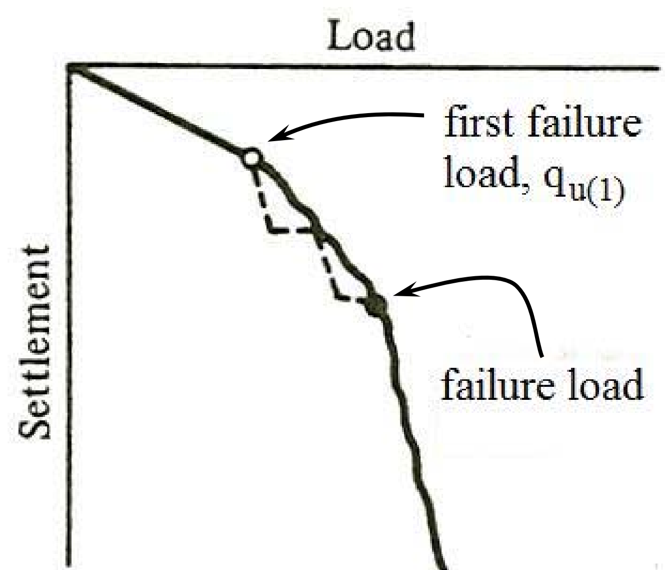


One sided rupture failure surface from a vertical, eccentric loading (Jumkins, 1956)

Local Shear Failure

- **Characterized by failure pattern clearly visible only immediately below footing**
- **Load-displacement curve does not show a clear peak**

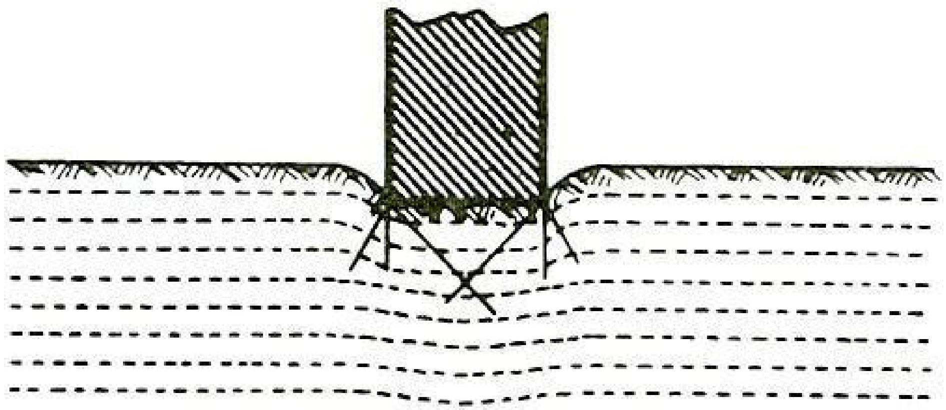




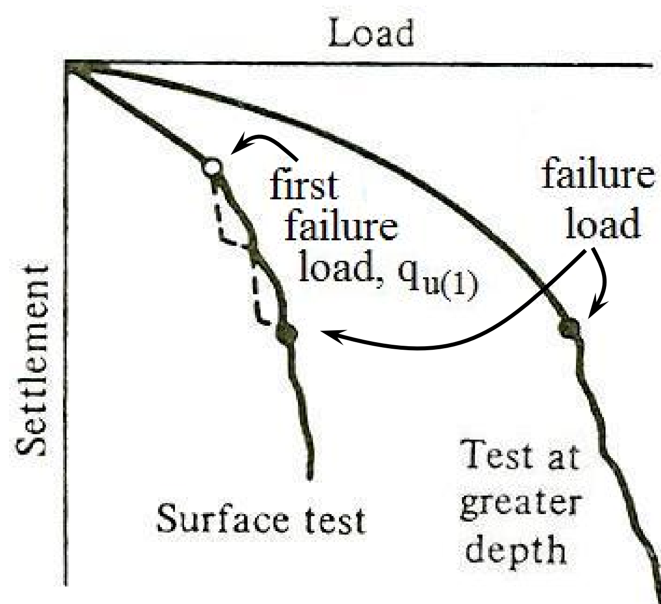
(Vesic, 1975)

Punching Shear Failure

- **Failure pattern is not easy to observe**
- **Compression of the soil immediately below footing occurs; no movement of soils on the sides**
- **Jerks and sudden movements in the vertical dir.**



(Vesic, 1975)

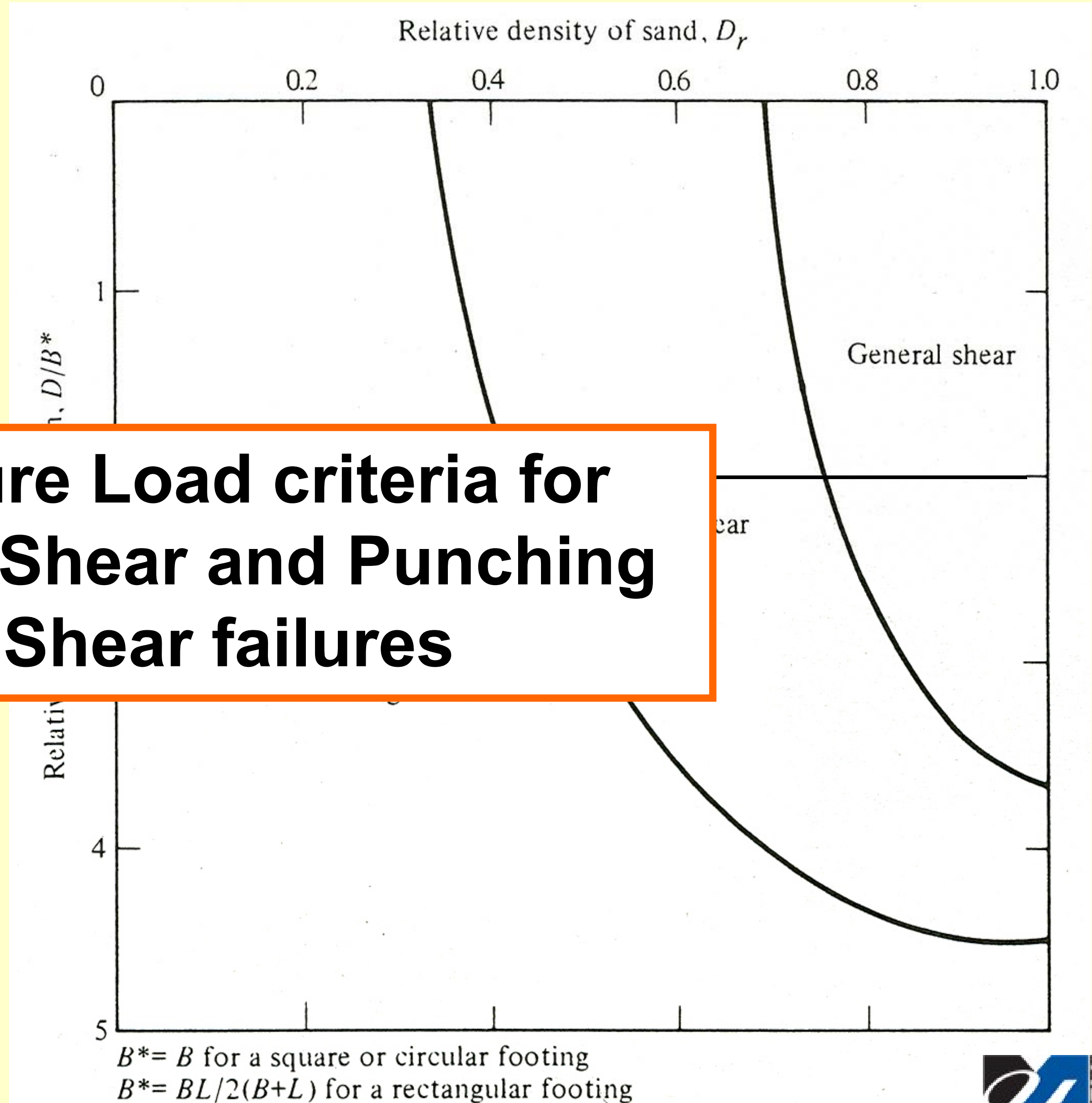


Modes of Failure and Relative Density

Generally speaking, a general shear failure takes place if the soil is incompressible and punching shear failure if compressib…

Failure mo… depends on embedment ratio and loading type

(Vesic, 1963 modified by De Beer, 1970)



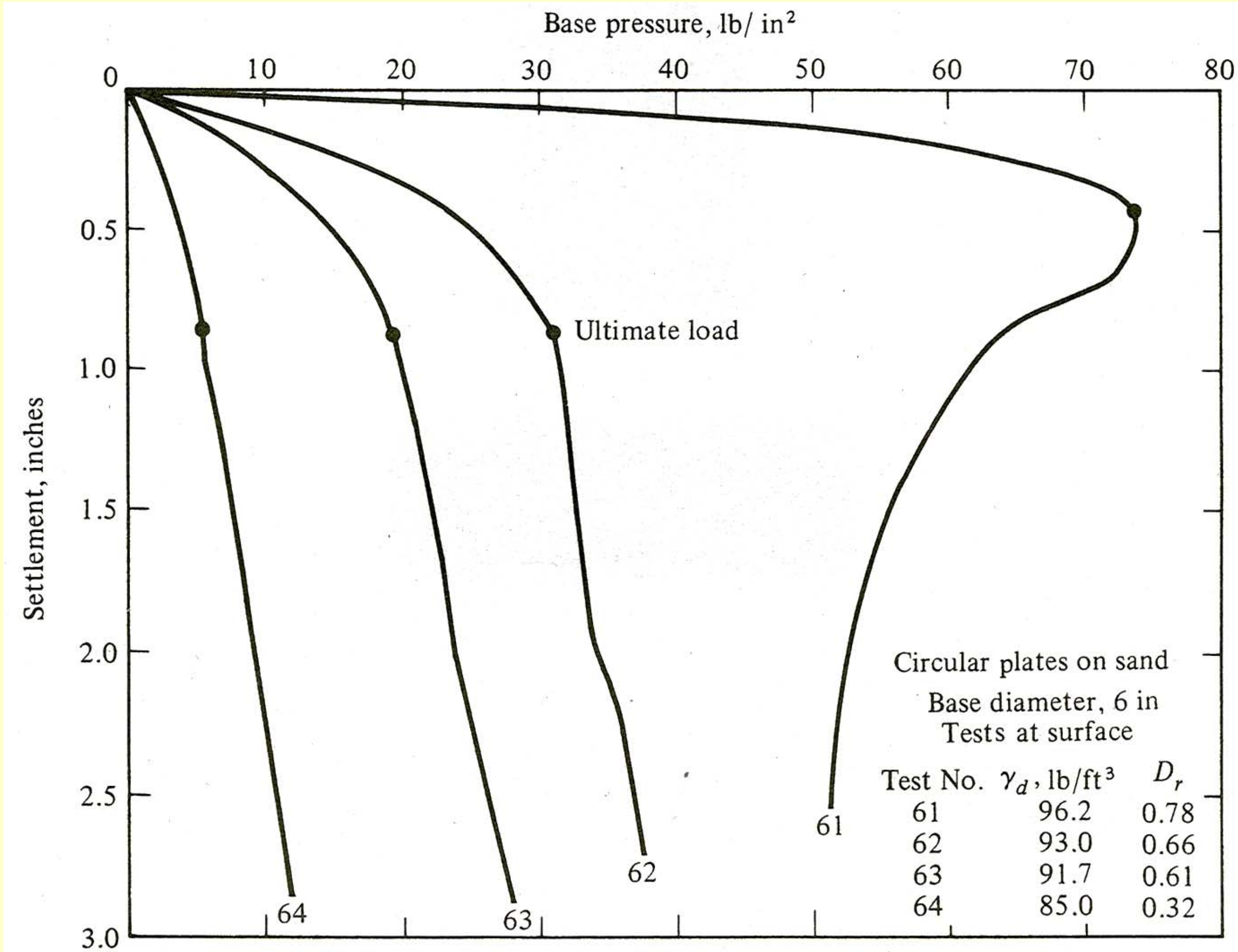
Failure Load criteria for Local Shear and Punching Shear failures

Failure (Ultimate Load) Criteria

- Minimum Slope Failure Load Criterion (Vesic, 1963)
- Limited Settlement Criterion of 0.1B (Vesic, 1975)
- Log-log Plot of Load-settlement Curve (De Beer, 1967)
- Two-slope Criterion
- Recommended Criterion
- Failure Interpretation Examples in Soils and Rocks

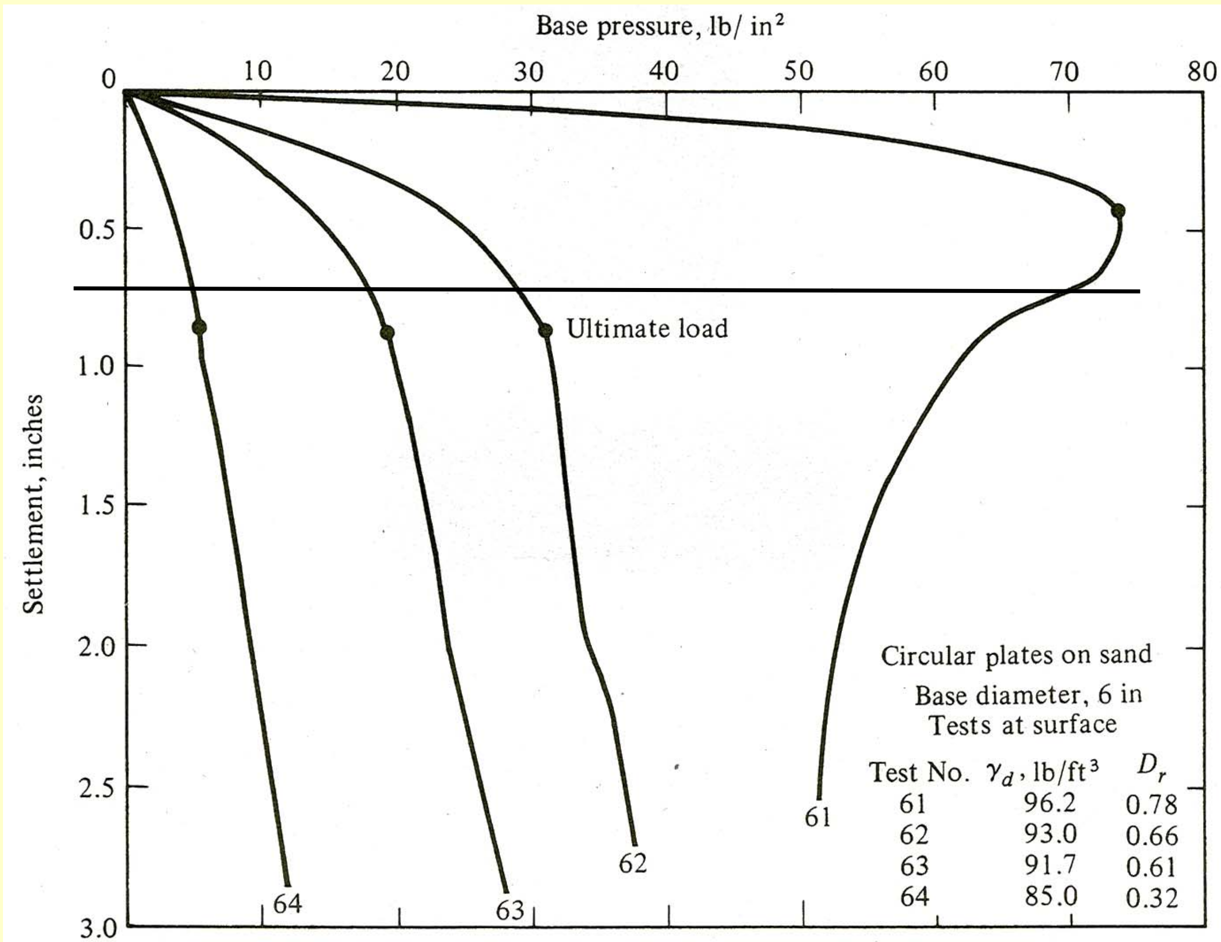
Minimum Slope Failure Criteria, Vesic (1963)

- **Ultimate Load is the point where the slope of the curve first reaches a steady, minimum value or zero**



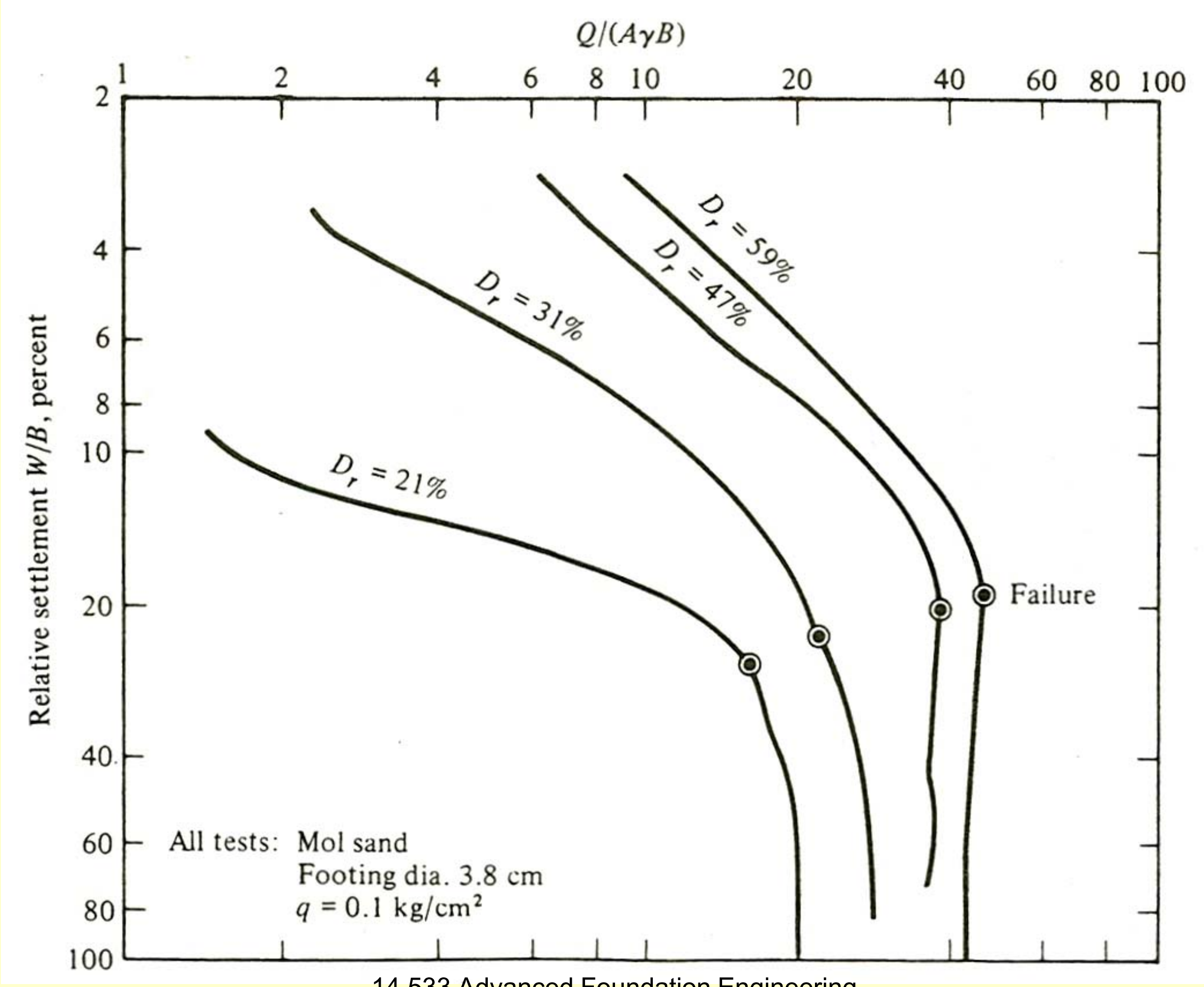
10% Width Settlement Criterion, Vesic (1975)

- **Cases in which minimum slope on the curve cannot be established with certainty**
- **Conservative estimates and may be problematic for larger foundations**



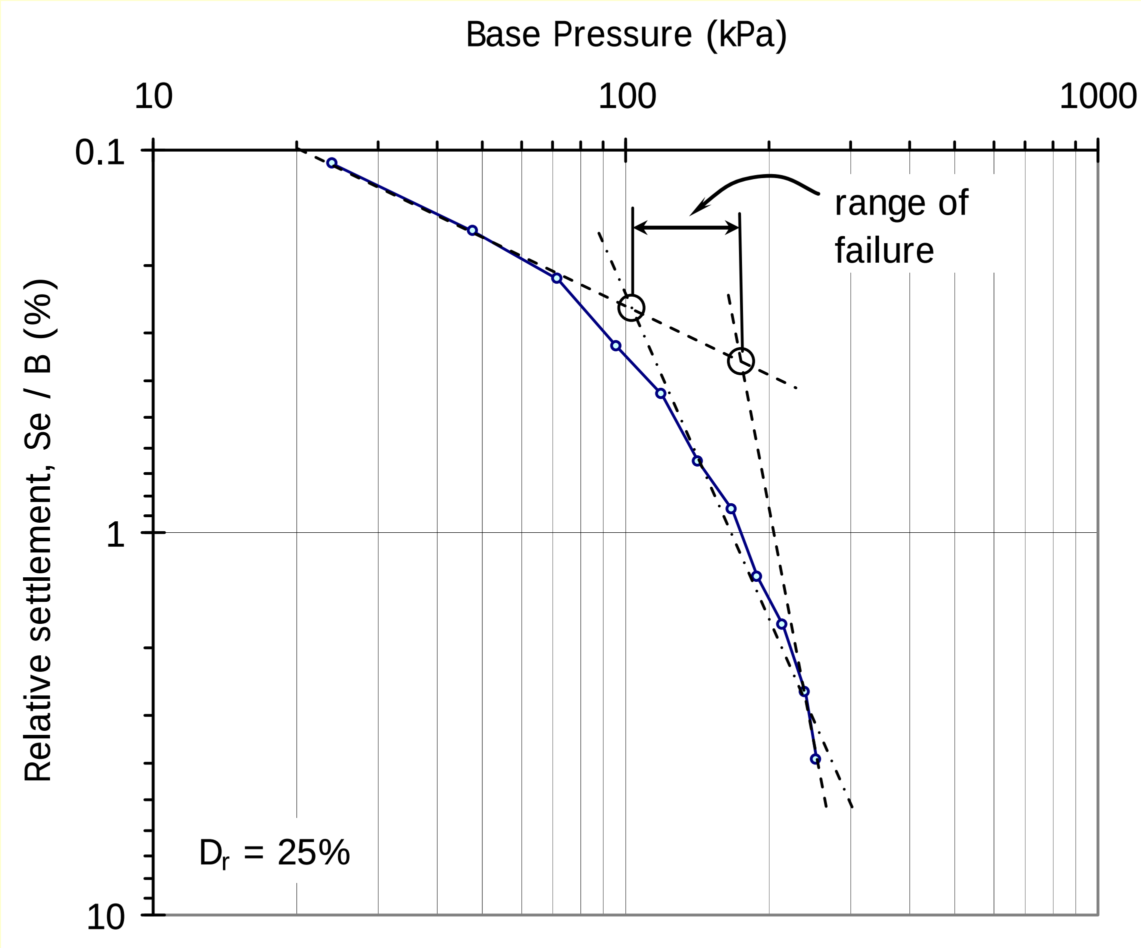
Log-log plot of Load-Settlement, De Beer (1967)

- **Ultimate Load defined as the change in load-settlement curve as the point of break of the curve (Circled Dots)**
- **Found to be very conservative compared to Minimum Slope**



Two Slope Criterion

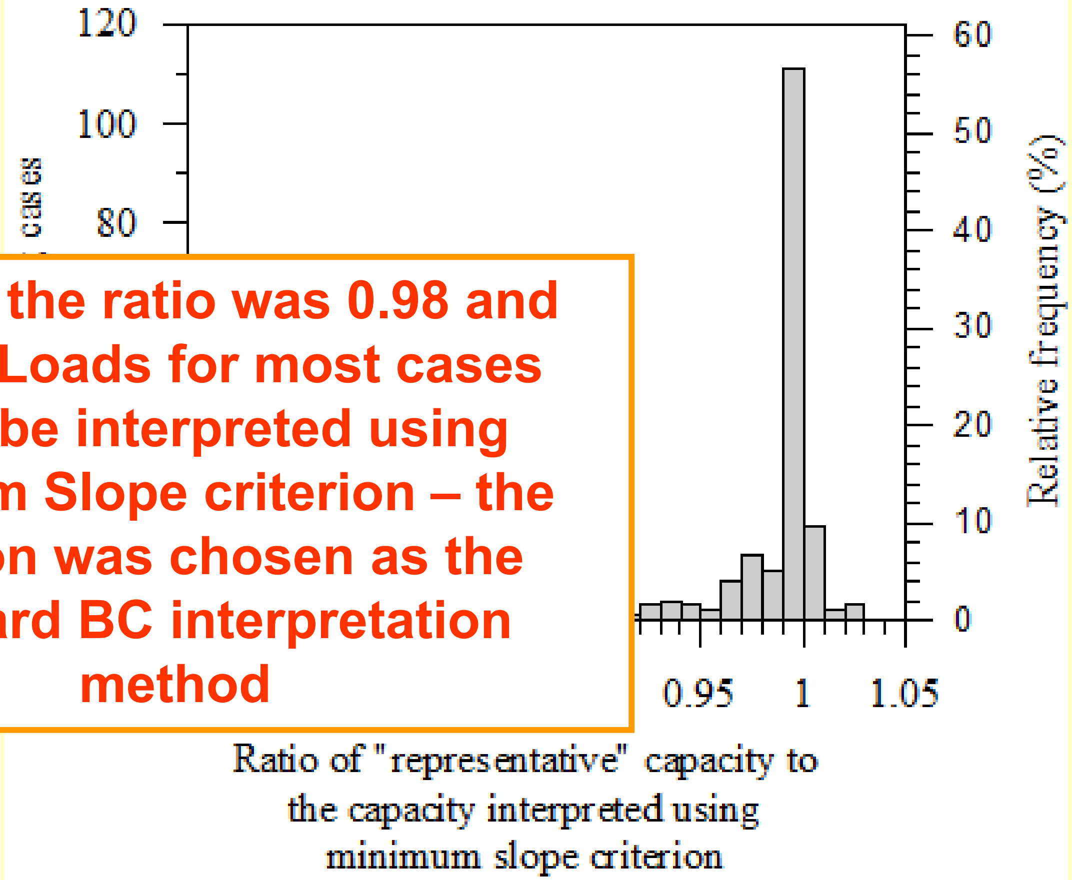
- **Ultimate Load taken as the intersection of the two asymptotes to the curve at the beginning and the end of loading test**
- **Sometimes a range of loads is possible; take mean value**



Recommended Failure Criterion

Minimum Slope Failure Load Criterion, Vesic (1963)

- **Failure load interpreted were for 196 cases using each of the proposed methods**
- **"Representative Failure Load" defined as the mean value of all the failure loads interpreted using each criterion**



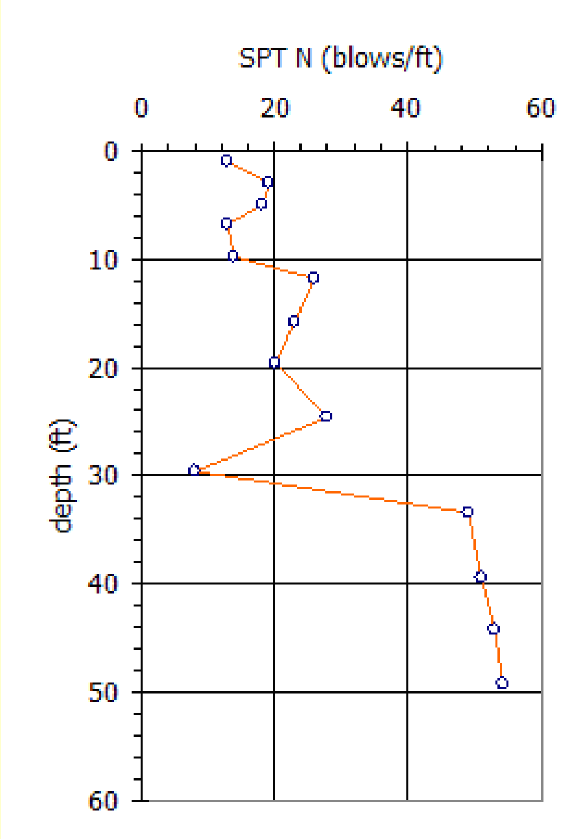
Mean of the ratio was 0.98 and Failure Loads for most cases could be interpreted using Minimum Slope criterion – the criterion was chosen as the standard BC interpretation method

Interpreted Failure Load – Example

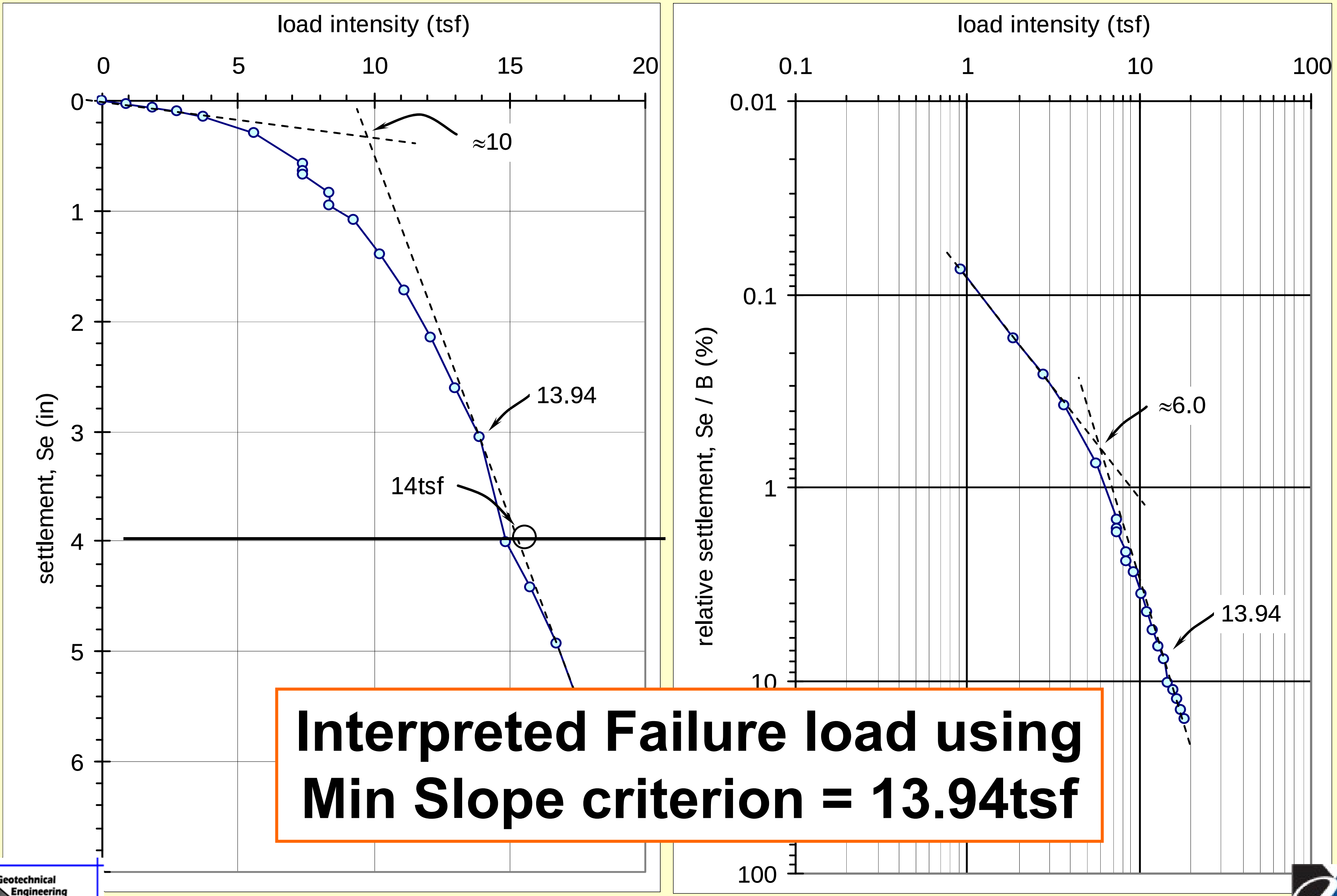
FOTID #35 (TAMU, Riverside Campus)

- **Reported by Briaud and Gibbens (1994) in Geotechnical Special Publication No. 41 (ASCE)**

- **39in x 39in square footing with 28in embedment**
- **Soil information**
  - **Ground level – 11.5ft: silty fine sand**
  - **11.5ft – 23ft: med dense silty sand**
  - **GWT at 16.0ft**
- **SPT-N counts as shown**
- **Average soil unit weight = 118pcf**
- **Average relative density = 50.75%**



Interpreted Failure Load – Example



Interpreted Failure load using Min Slope criterion = 13.94tsf

Uncertainty Evaluation – Granular Soils

BC of centric vertically loaded footings

- **Database Overview**
- **Calculated Bearing Capacity**
  - **Soil Parameters**
  - **Equations used for BC calculations**

Database – Overview

UML-GTR ShalFound07

Foundation type	Predominant Soil Type					Total	Country	
	Sand	Gravel	Cohesive	Mix	Others		Germany	Others
Plate load tests B ≤1m	346	46	--	2	72	**466**	253	213
Small footings 1 < B ≤ 3m	26	2	--	4	1	**33**	--	33
Large footings 3 < B ≤ 6m	30	--	--	1	--	**31**	--	31
Rafts & Mats B > 6m	13	--	--	5	1	**19**	1	18
Total	**415**	**48**	**0**	**12**	**74**	**549**	**254**	**295**

Note:

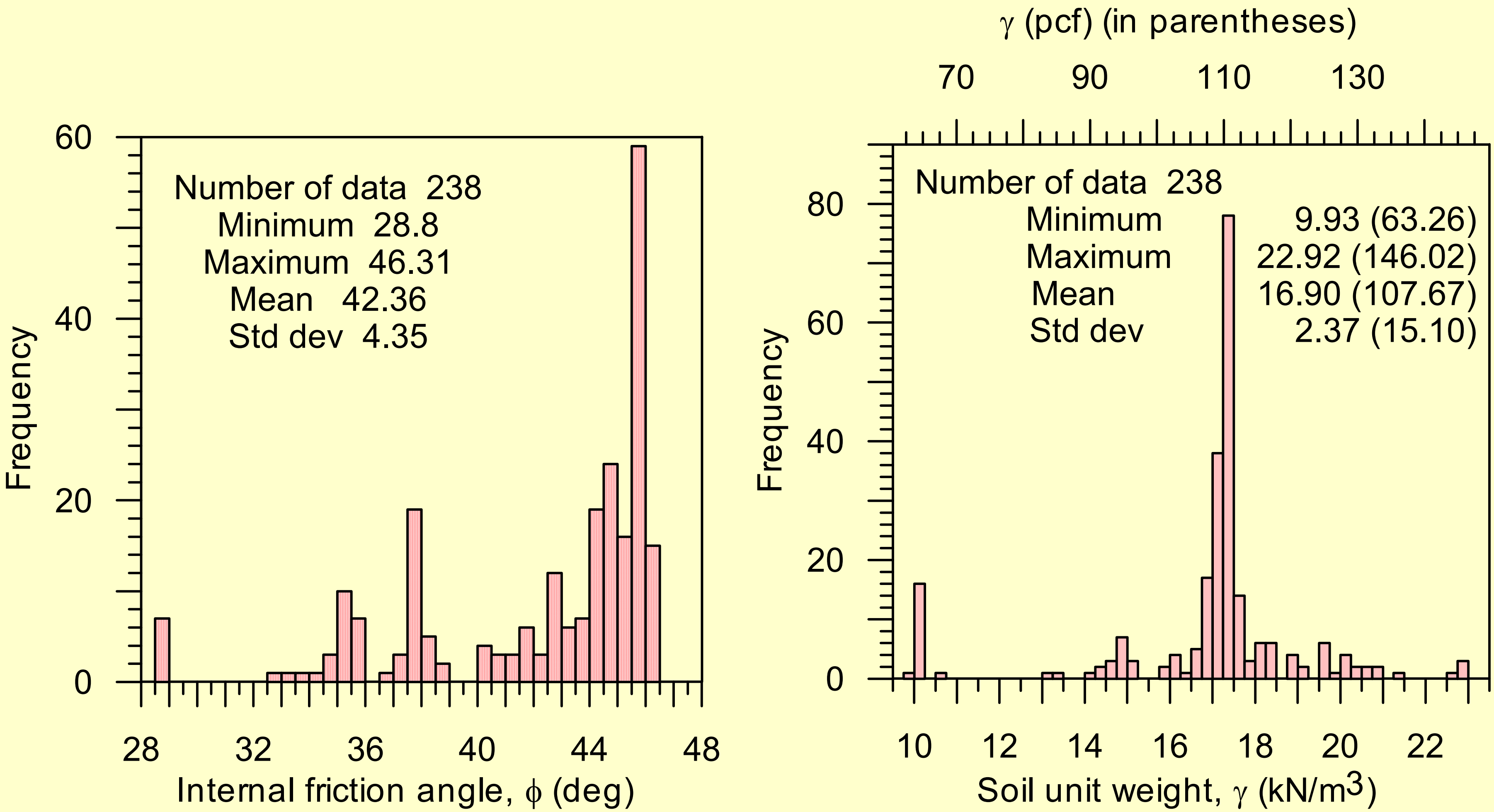
"Mixed" are cases with alternating layers of sand or gravel and clay or silt

"Others" are cases with either unknown soil types or with other granular materials like loamy Scoria

1m ≈ 3.3ft

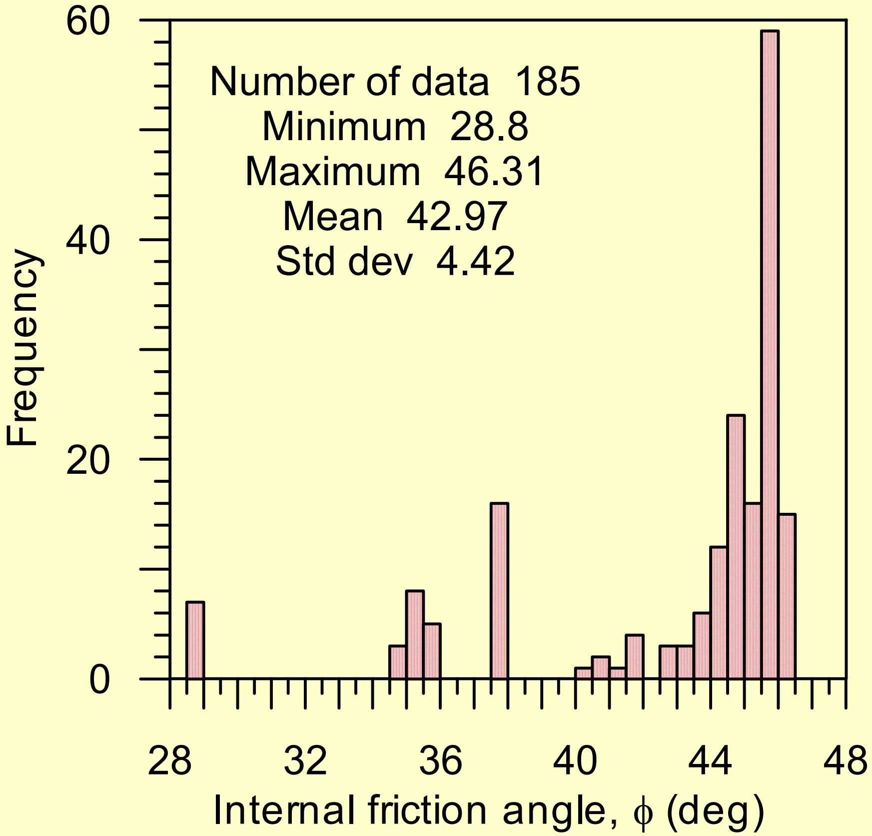
Database – Overview

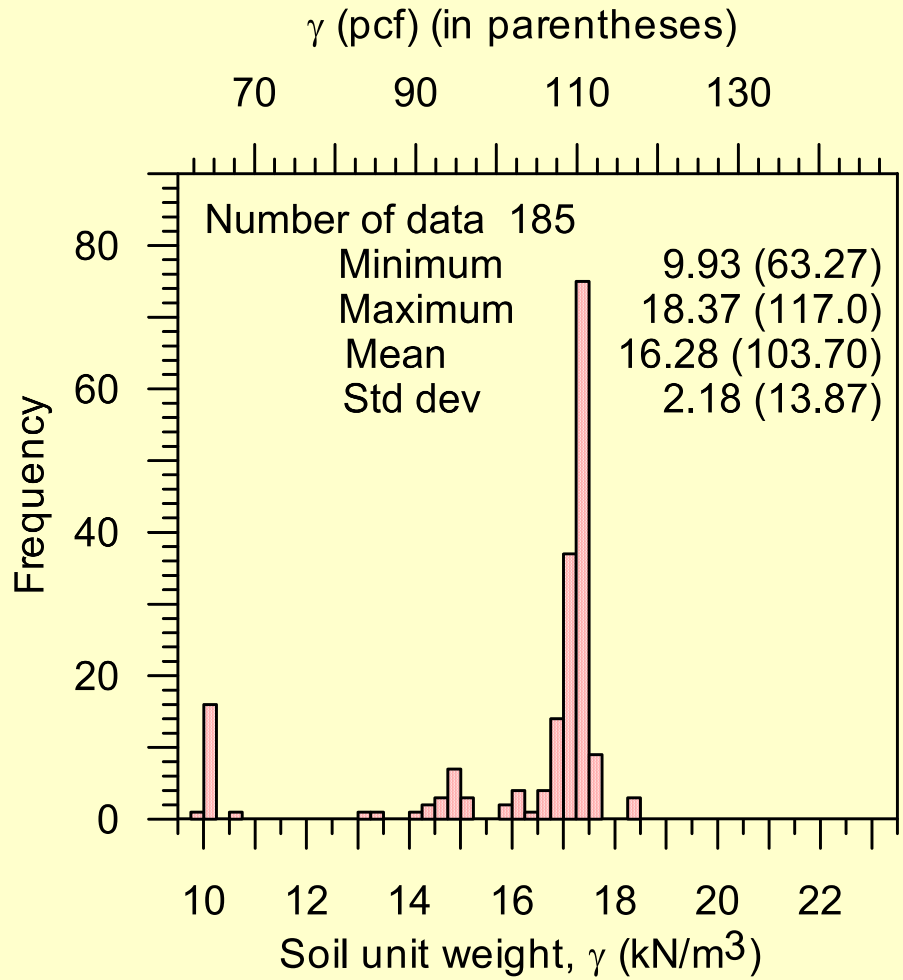
UML-GTR ShalFound07: Database I



Database – Overview

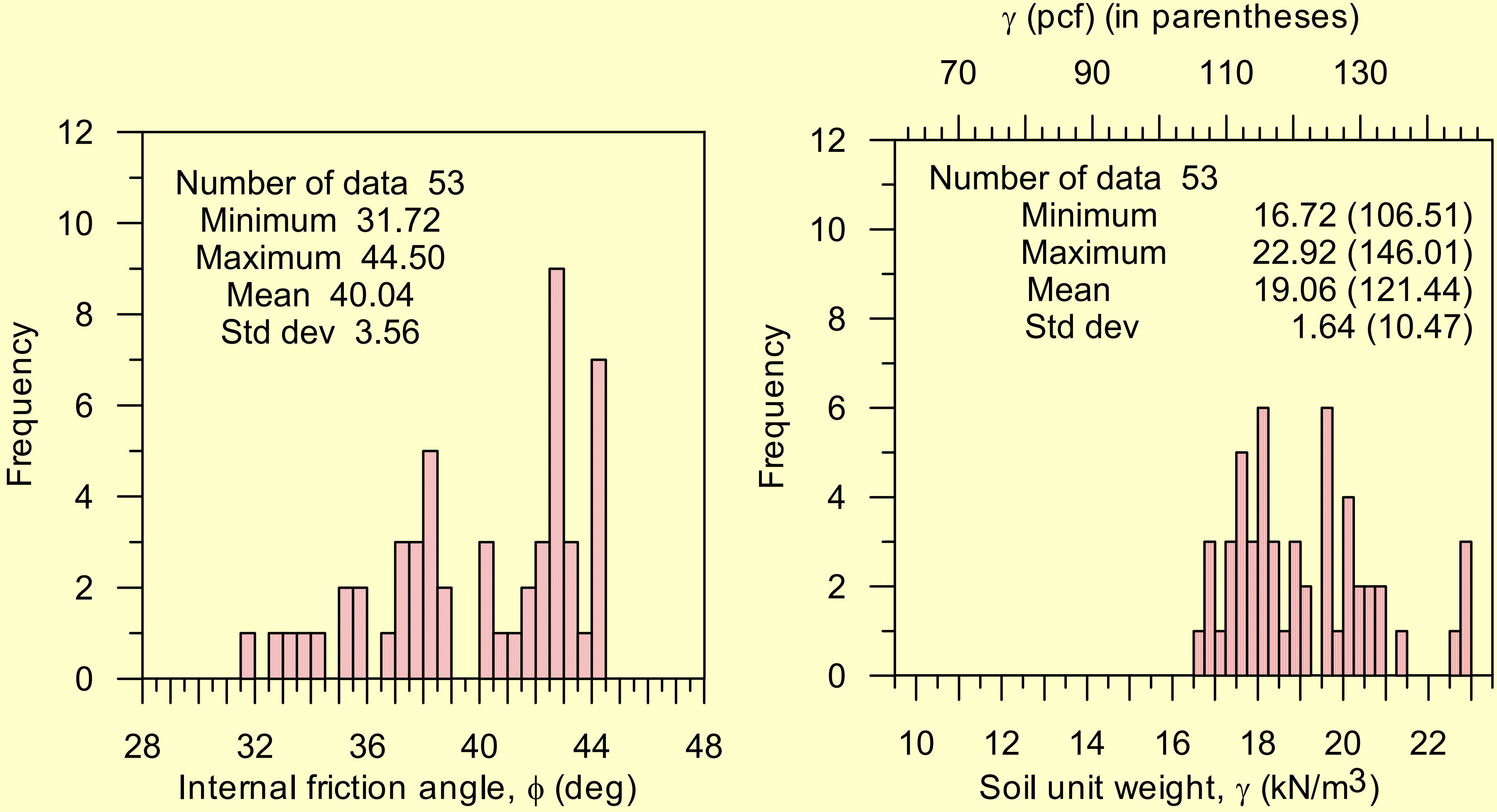
Controlled Soil Conditions





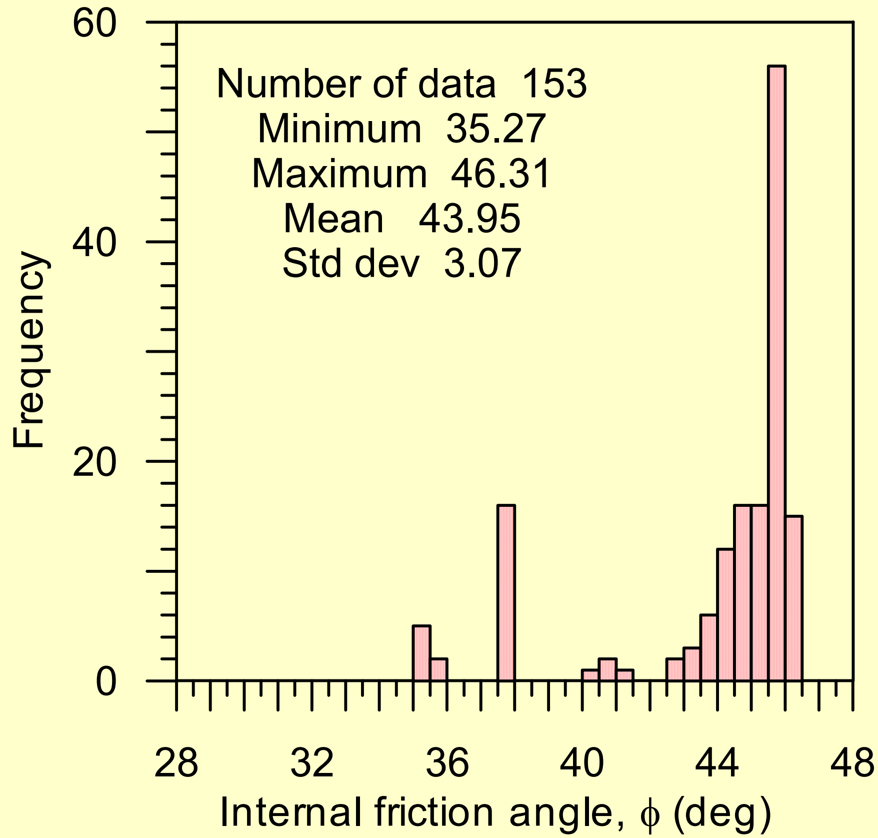
Database – Overview

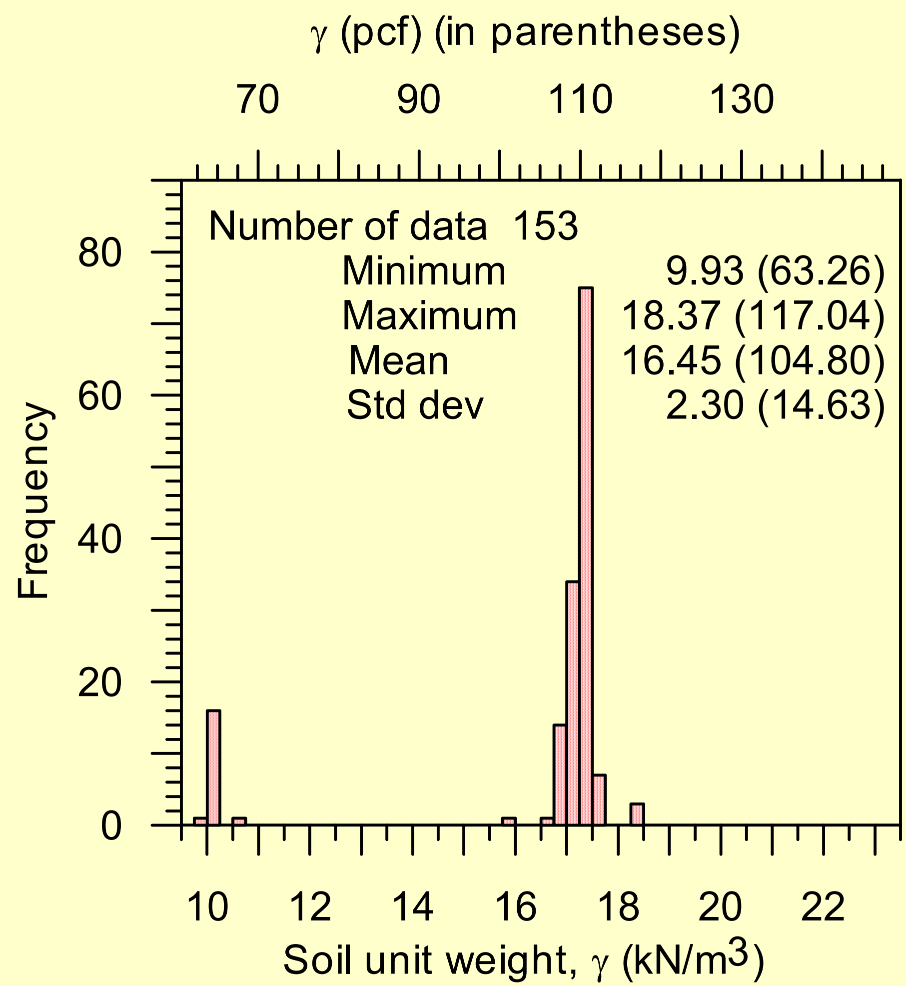
Natural soil conditions



UML-GTR ShalFound07: Database I

Cases in/on granular soils – German tests





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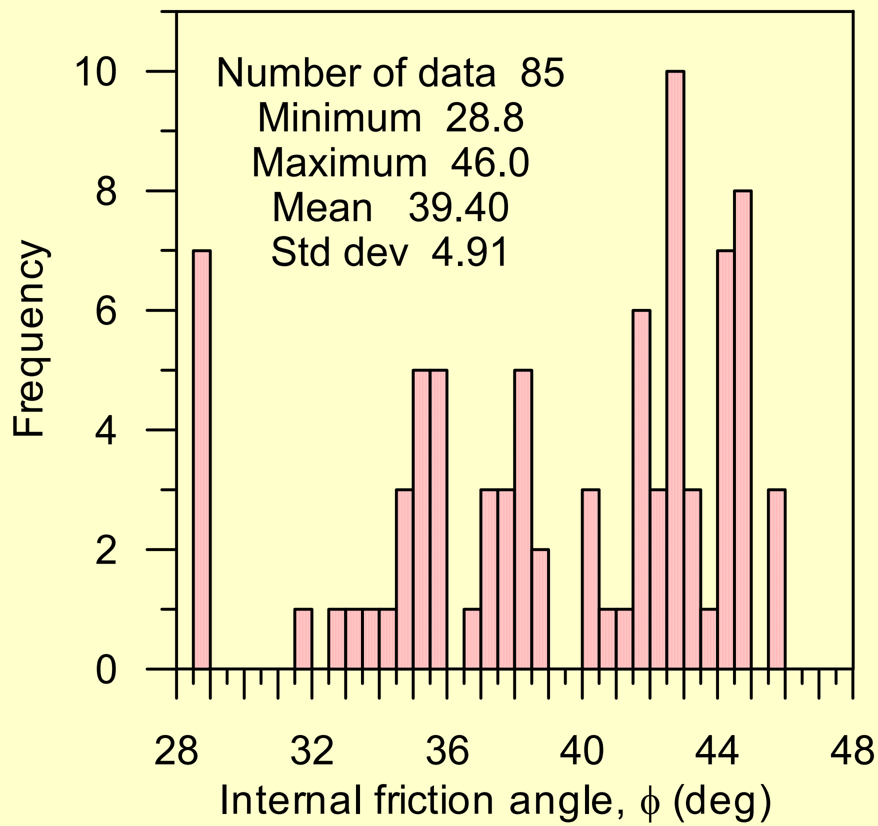
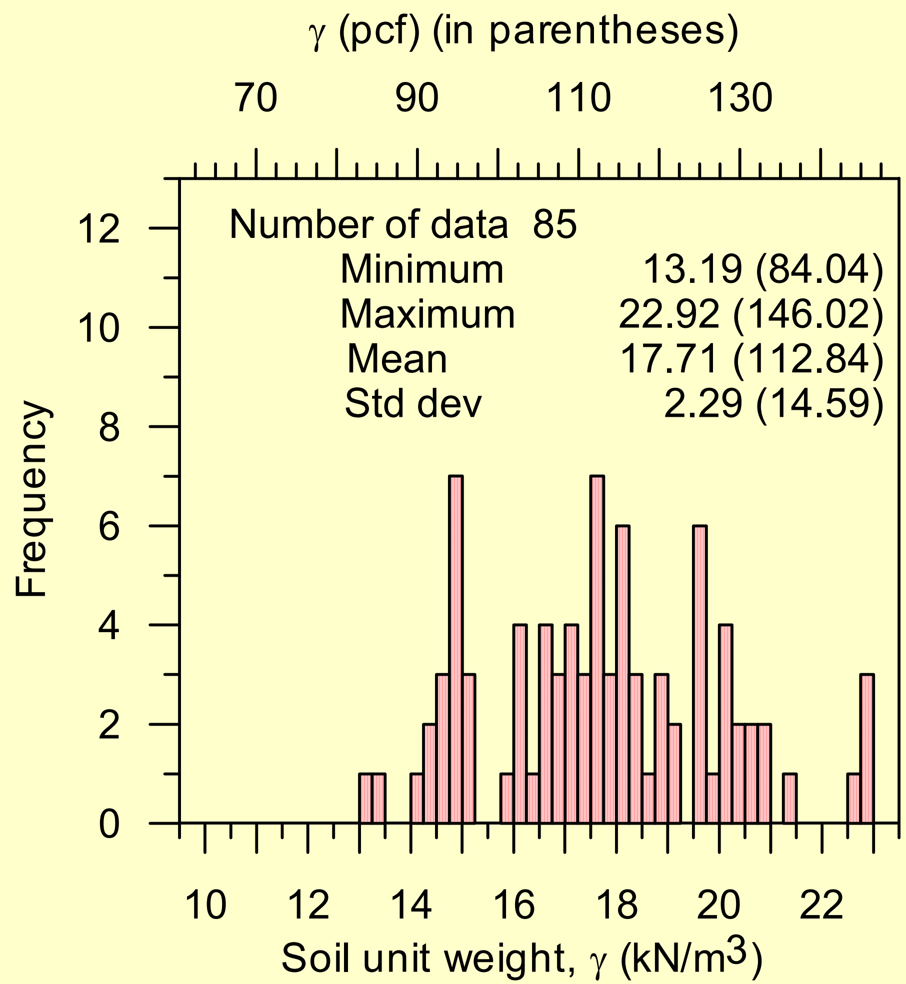
UML-GTR ShalFound07: Database I

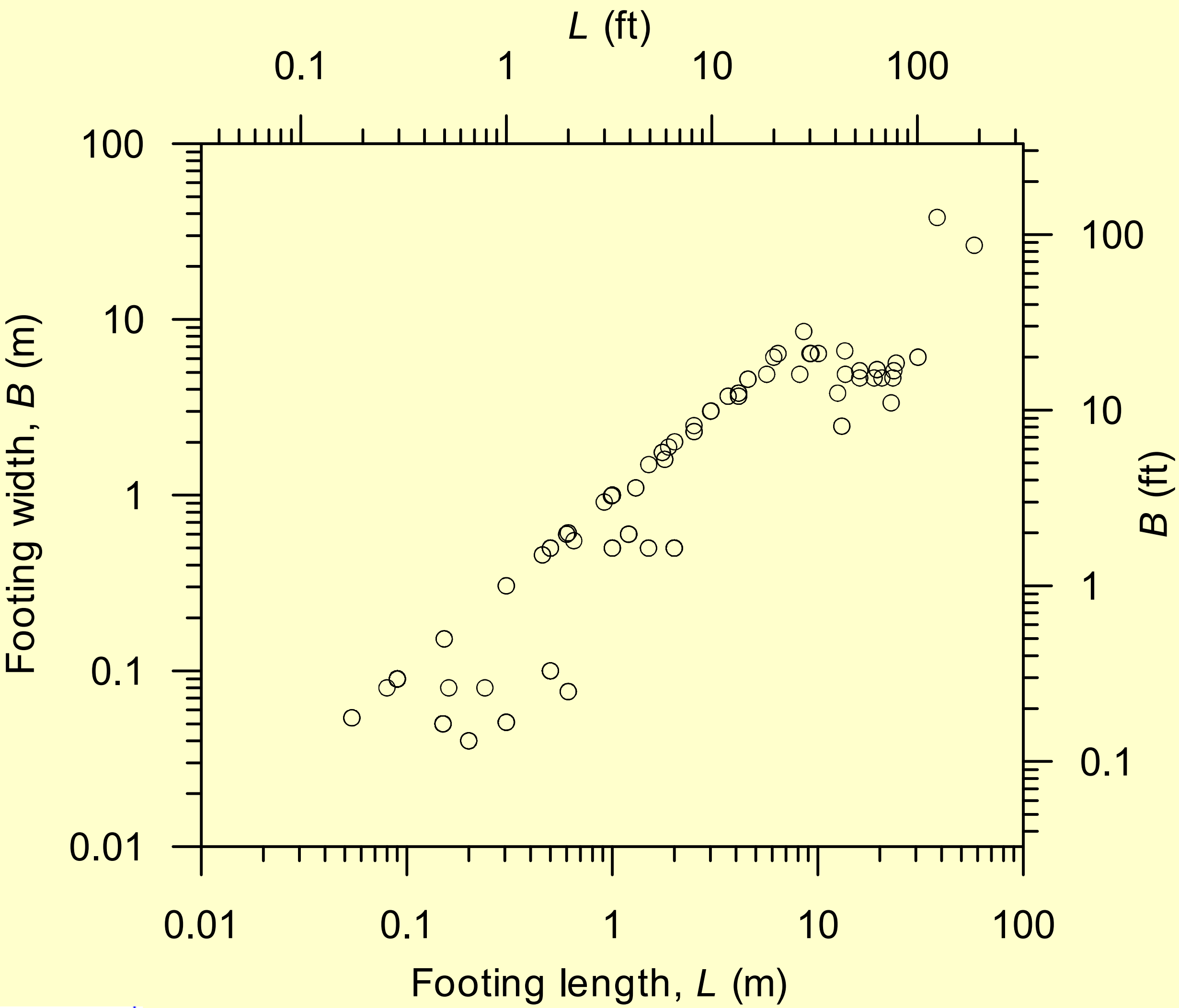
Cases in/on granular soils – Non-German tests





Database – Overview

Footing sizes



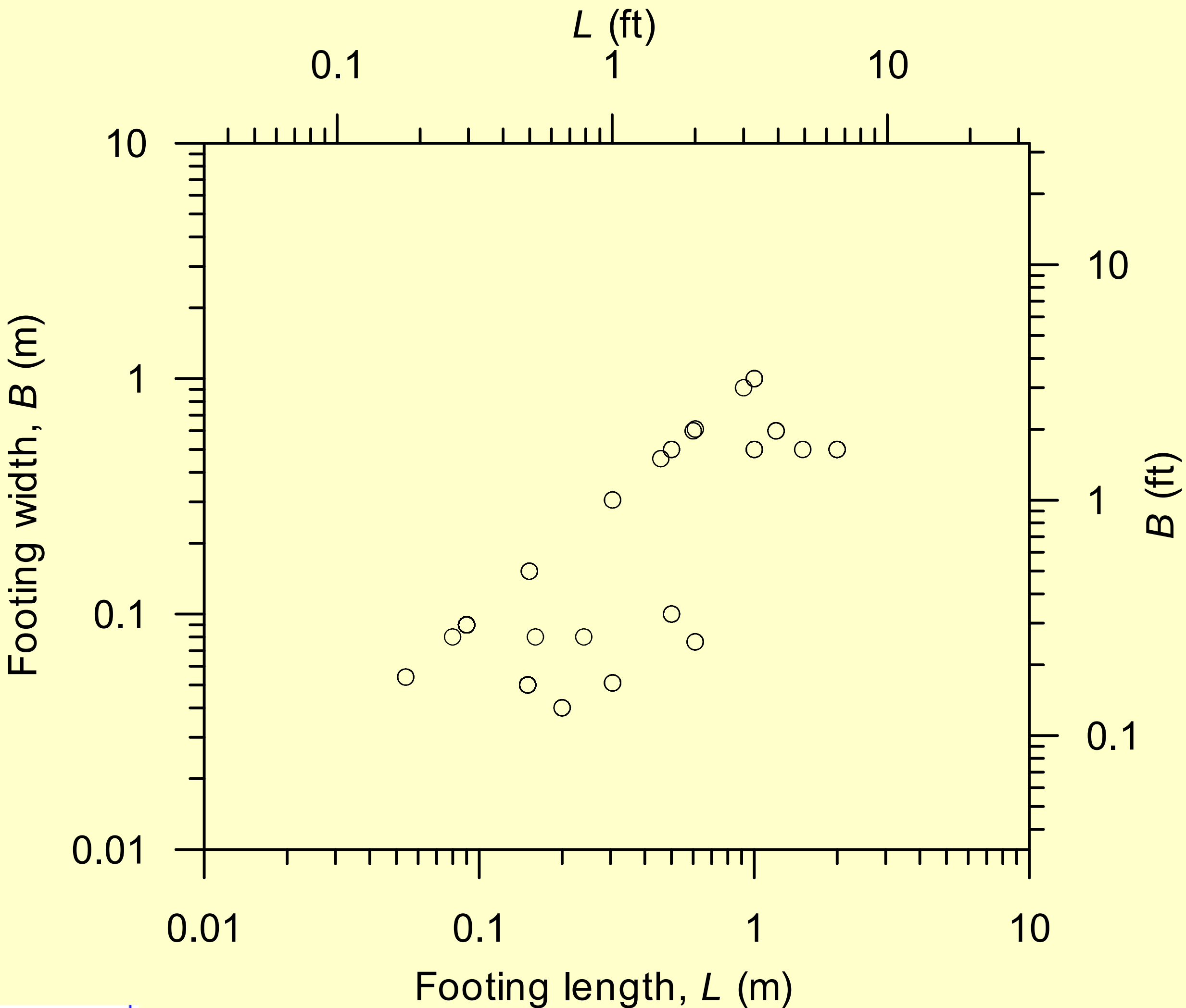
Number of data 238

Width in m (ft)
Minimum 0.04 (0.13)
Maximum 38.1 (125.0)
Mean 1.25 (4.10)
Median 0.09 (0.30)
Std dev 3.40 (11.15)

Length in m (ft)
Minimum 0.054 (0.177)
Maximum 57.7 (189.3)
Mean 2.62 (8.56)
Median 0.15 (0.49)
Std dev 6.80 (22.34)

UML-GTR ShalFound07: Database I

Footing sizes – Controlled soil conditions



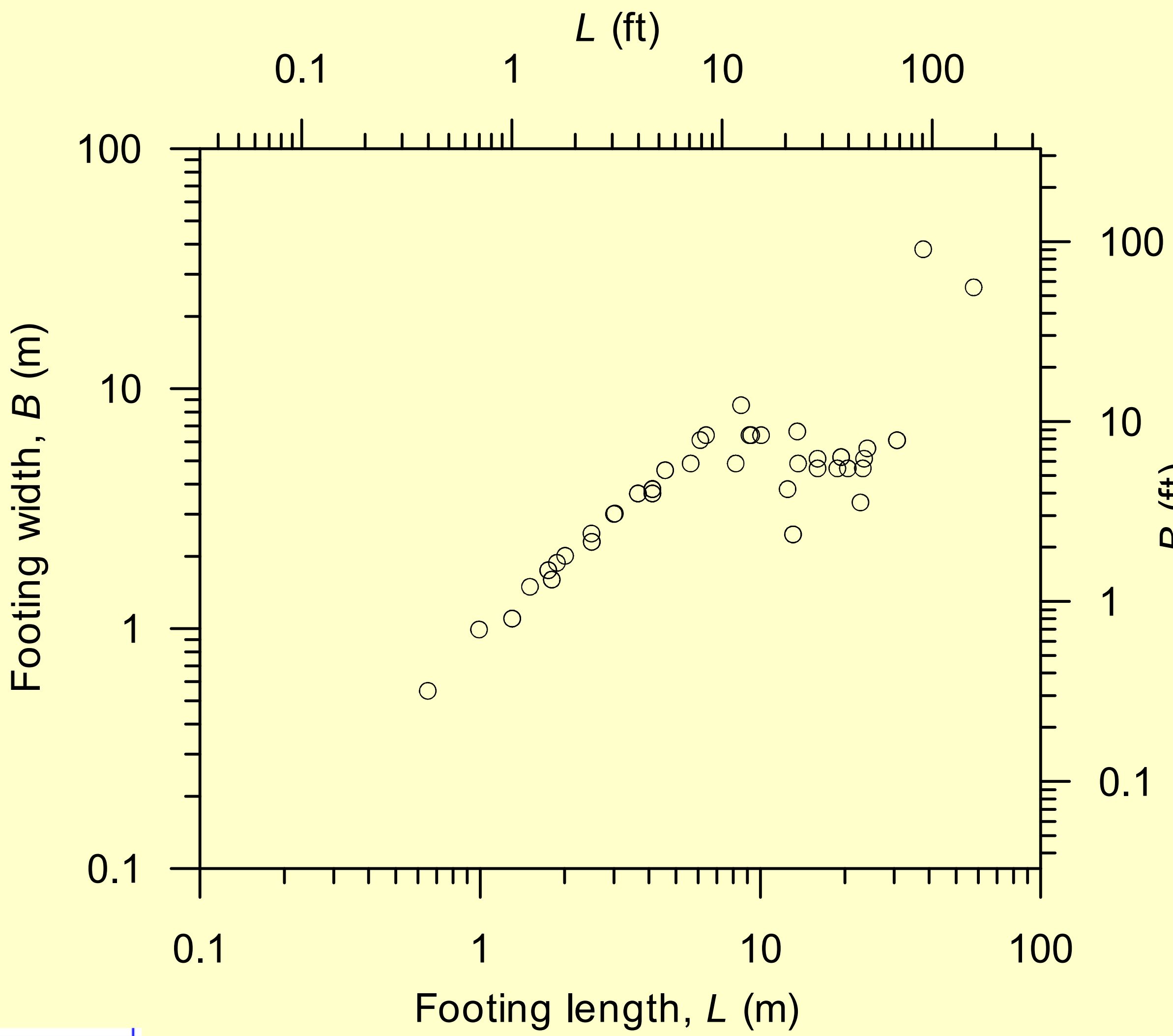
Number of data 185

Width in m (ft)
Minimum 0.04 (0.13)
Maximum 1.00 (3.28)
Mean 0.18 (0.60)
Median 0.09 (0.30)
Std dev 0.22 (0.73)

Length in m (ft)
Minimum 0.054 (0.177)
Maximum 2.0 (6.56)
Mean 0.33 (1.07)
Median 0.09 (0.30)
Std dev 0.46 (1.50)

UML-GTR ShalFound07: Database I

Footing sizes – Natural soil conditions



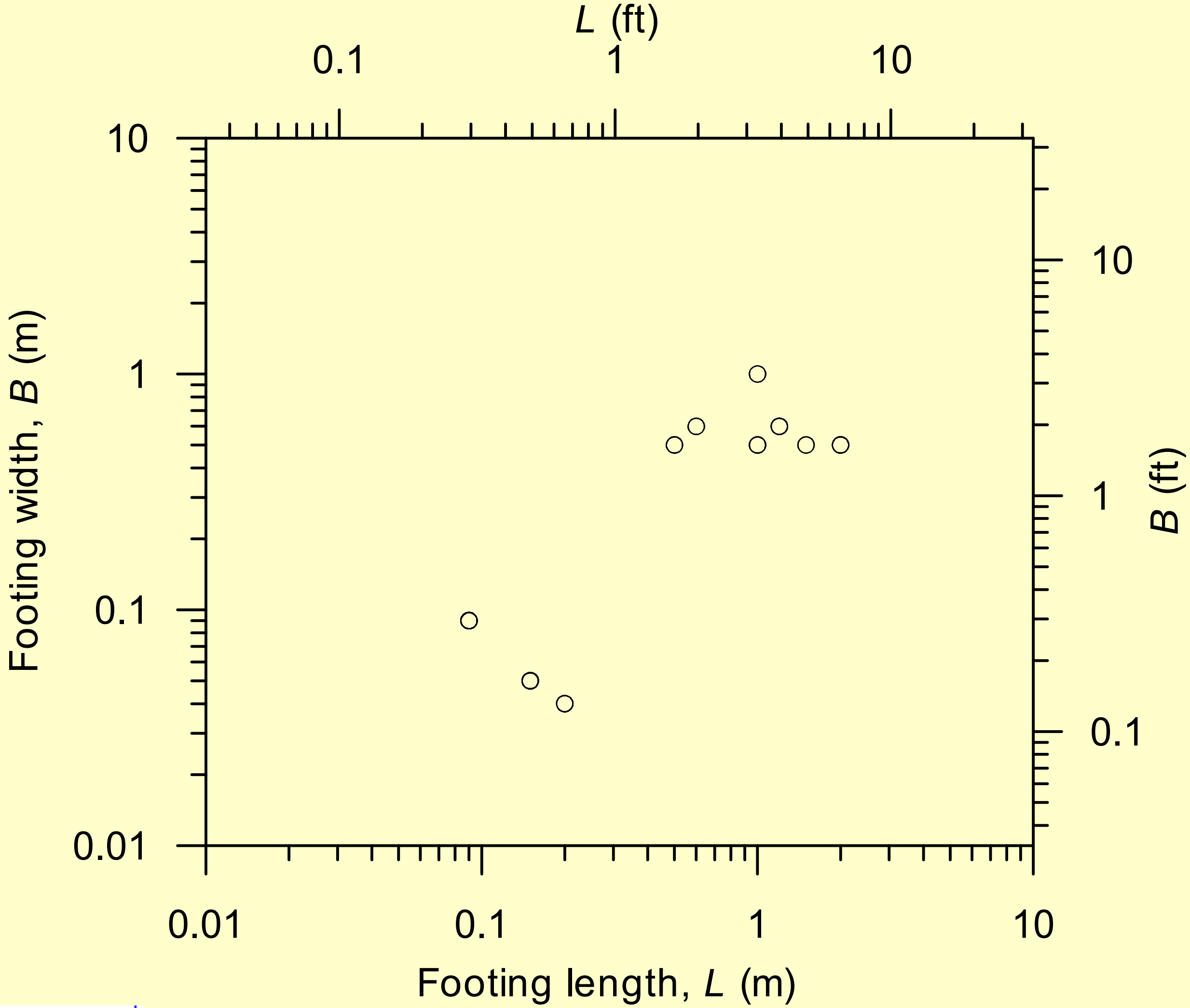
Number of data 53

Width in m (ft)
Minimum 0.55 (1.80)
Maximum 38.1 (125.0)
Mean 4.96 (16.27)
Median 3.81 (12.50)
Std dev 5.86 (19.23)

Length in m (ft)
Minimum 0.65 (2.13)
Maximum 57.7 (189.3)
Mean 10.59 (34.74)
Median 5.64 (18.50)
Std dev 11.24 (36.88)

UML-GTR ShalFound07: Database I

Cases in/on granular soils – German tests



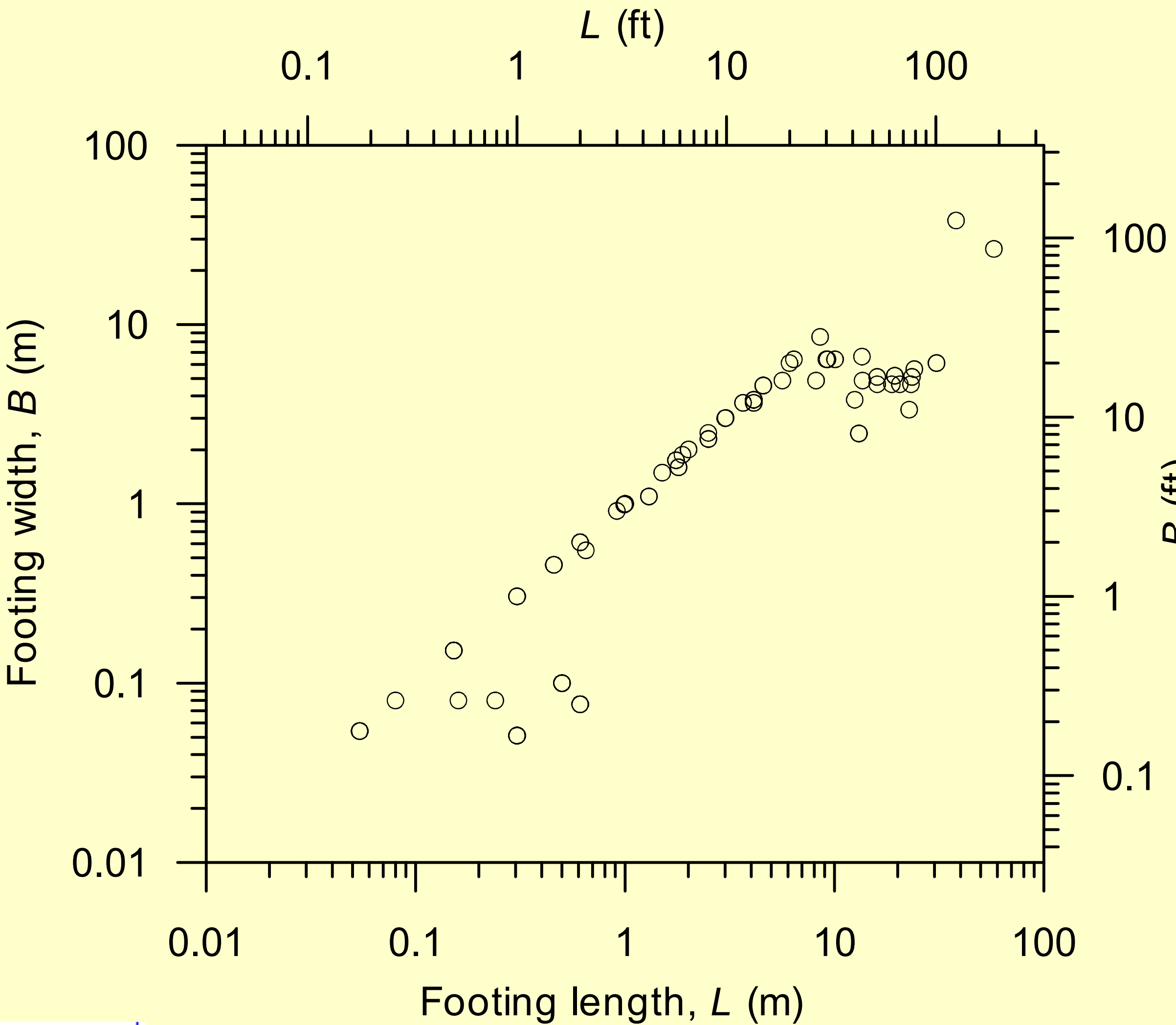
Number of data 153

Width in m (ft)
Minimum 0.04 (0.13)
Maximum 1.0 (3.28)
Mean 0.16 (0.53)
Median 0.09 (0.30)
Std dev 0.20 (0.66)

Length in m (ft)
Minimum 0.09 (0.30)
Maximum 2.0 (6.56)
Mean 0.31(1.01)
Median 0.09 (0.30)
Std dev 0.48 (1.59)

UML-GTR ShalFound07: Database I

Cases in/on granular soils – Non-German tests



Number of data 85

Width in m (ft)
Minimum 0.051 (0.17)
Maximum 38.1 (125.0)
Mean 3.20 (10.50)
Median 2.01 (7.0)
Std dev 5.15 (16.90)

Length in m (ft)
Minimum 0.054 (0.18)
Maximum 57.7 (189.3)
Mean 6.76 (22.18)
Median 2.01 (7.0)
Std dev 10.14 (33.3)

Equations used for BC calculation

The bearing capacity equation specified in AASHTO (2008) with minimal necessary adjustment has been used to calculate the bearing capacity of a footing of length L and width B′ and supported by a soil with cohesion c, average friction angle ϕ_f and average unit weights γ_1 and γ_2 above and below the footing base, respectively. The format presented in equation (95) is based on the general bearing capacity formulation used by Vesić (1975) as presented in section 1.5.3 equation (34). The numbering in the parenthesis represents the proposed numbering for the modified AASHTO specifications.

$$q_n = c \cdot N_{cm} + \gamma_1 \cdot D_f \cdot N_{qm} + 0.5 \cdot \gamma_2 \cdot B \cdot N_{\gamma m} \quad (10.6.3.1.3a\text{-}1) \quad (95)$$

In which:

$$N_{cm} = N_c s_c d_c i_c \quad (10.6.3.1.3a\text{-}2) \quad (96)$$

$$N_{qm} = N_q s_q d_q i_q \quad (10.6.3.1.3a\text{-}3) \quad (97)$$

$$N_{\gamma m} = N_\gamma s_\gamma d_\gamma i_\gamma \quad (10.6.3.1.3a\text{-}4) \quad (98)$$

where:

- c = cohesion, taken as undrained shear strength c_u in total stress analysis or as cohesion c' in effective stress analysis (ksf)
- N_c = cohesion term bearing capacity factor as specified in Tables 25 and 26 (dim.)
- N_q = surcharge (embedment) term bearing capacity factor as specified in Tables 25 and 26 (dim.)
- N_γ = unit weight (footing width) term bearing capacity factor as specified in Tables 25 and 26 (dim.)
- γ_1 = moist or submerged unit weight of soil above the bearing depth of the footing (kcf)
- γ_2 = moist or submerged unit weight of soil below the bearing depth of the footing (kcf)
- D_f = footing embedment depth (ft.)
- B = footing width (ft.), equal to the physical footing width B in case of centric loading or effective footing width B' in case of eccentric loading
- s_c, s_γ, s_q = footing shape correction factors as specified in Table 27 (di.)
- d_c, d_γ,, d_q = depth correction factors to account for the shearing resistance along the failure surface passing through the soil above the bearing elevation as specified in Table 28 (dim.)
- i_c, i_γ, i_q = load inclination factors as specified in Table 29 (dim.).

Equations used for BC calculation

The effective vertical stress calculated at the base of the footing ($\sum_{0}^{D_f} \gamma_i D_i$) should be used or alternatively, an average weighted soil unit weight ($\gamma_{1,ave}$) should be used above the base. Below the base an average soil unit weight ($\gamma_{2,ave}$) should be used within a zone of 1.5B. The highest anticipated groundwater level should be used in design.

In Tables 27 to 29 B and L are either the physical footing dimensions in case of centric loading or have to be substituted by the effective footing dimensions B' and L' in case of eccentric loading.

In Table 29 H and V are the unfactored horizontal and vertical loads, in (kips), respectively. The angle θ is the projected direction of load in the plane of the footing, measured from the side of the footing length *L* (deg.). Figure 17 (AASHTO Figure 10.6.3.1.3a-1) shows the conventions for determining θ. The parameter n is defined according to equation (99).

$$n = \left[\frac{(2 + L'/B')}{(1 + L'/B')}\right]\cos^2\theta + \left[\frac{(2 + B'/L')}{(1 + B'/L')}\right]\sin^2\theta \qquad (10.6.3.1.3a\text{-}5) \qquad (99)$$

Equations used for BC calculation

The depth correction factor should be used only when the soils above the footing bearing elevation are competent and there is no danger for their removal over the foundation's lifetime, otherwise, the depth correction factor should be taken as 1.0, or D_f should be reduced to include the competent, secured depth only.

The depth correction factors presented in Table 28 refers when applicable to the effective foundation width B′. Some design practices use the physical footing width (B) for evaluating the depth factors under eccentric loading as well. The calibration presented in this study was conducted using B′. The use of B in the depth factors expressions, results with a more conservative evaluation as discussed by Paikowsky et al. (2009a).

Table 25 Bearing capacity factors N_c (Prandtl, 1921), N_q (Reissner, 1924), and N_γ (Vesic, 1975) (AASHTO Table 10.6.3.1.3a-1)

Factor	Friction Angle	Cohesion Term (N_c)	Unit Weight Term (N_γ)	Surcharge Term (N_q)
Bearing Capacity Factors N_c, N_γ, N_q	$\phi_f = 0$	$2 + \pi$	0.0	1.0
	$\phi_f > 0$	$(N_q - 1)\cdot\cot\phi_f$	$2\cdot(N_q + 1)\cdot\tan\phi_f$	$\exp(\pi\cdot\tan\phi_f)\cdot\tan^2\left(45+\frac{\phi_f}{2}\right)$

Equations used for BC calculation

Table 26 Bearing capacity factors N_c (Prandtl, 1921), N_q (Reissner, 1924), and N_γ (Vesic, 1975) (AASHTO Table 10.6.3.1.3a-2)

ϕ_f	N_c	N_q	N_γ	ϕ_f	N_c	N_q	N_γ
0	5.14	1.0	0.0	23	18.1	8.7	8.2
1	5.4	1.1	0.1	24	19.3	9.6	9.4
2	5.6	1.2	0.2	25	20.7	10.7	10.9
3	5.9	1.3	0.2	26	22.3	11.9	12.5
4	6.2	1.4	0.3	27	23.9	13.2	14.5
5	6.5	1.6	0.5	28	25.8	14.7	16.7
6	6.8	1.7	0.6	29	27.9	16.4	19.3
7	7.2	1.9	0.7	30	30.1	18.4	22.4
8	7.5	2.1	0.9	31	32.7	20.6	26.0
9	7.9	2.3	1.0	32	35.5	23.2	30.2
10	8.4	2.5	1.2	33	38.6	26.1	35.2
11	8.8	2.7	1.4	34	42.2	29.4	41.1
12	9.3	3.0	1.7	35	46.1	33.3	48.0
13	9.8	3.3	2.0	36	50.6	37.8	56.3
14	10.4	3.6	2.3	37	55.6	42.9	66.2
15	11.0	3.9	2.7	38	61.4	48.9	78.0
16	11.6	4.3	3.1	39	67.9	56.0	92.3
17	12.3	4.8	3.5	40	75.3	64.2	109.4
18	13.1	5.3	4.1	41	83.9	73.9	130.2
19	13.9	5.8	4.7	42	93.7	85.4	155.6
20	14.8	6.4	5.4	43	105.1	99.0	186.5
21	15.8	7.1	6.2	44	118.4	115.3	224.6
22	16.9	7.8	7.1	45	133.9	134.9	271.8

Equations used for BC calculation

Table 27 Shape correction factors s_c, s_γ, s_q. (Vesić, 1975) (AASHTO Table 10.6.3.1.3a-3)

Factor	Friction Angle	Cohesion Term (s_c)	Unit Weight Term (s_γ)	Surcharge Term (s_q)
Shape Factors s_c, s_γ, s_q	$\phi_f = 0$	$1+0.2\cdot\frac{B}{L}$	1.0	1.0
	$\phi_f > 0$	$1+\frac{B}{L}\cdot\frac{N_q}{N_c}$	$1-0.4\cdot\frac{B}{L}$	$1+\frac{B}{L}\cdot\tan\phi_f$

Table 28 Depth correction factors d_c, d_γ, $_{dq}$. (Brinch Hansen, 1970) (AASHTO Table 10.6.3.1.3a-4)

Factor	Friction Angle	Cohesion Term (d_c)	Unit Weight Term (d_γ)	Surcharge Term (d_q)
Depth Correction Factors d_c, d_γ, d_q	$\phi_f = 0$	for $D_f \le B$: $1+0.4\cdot\frac{D_f}{B}$ for $D_f > B$: $1+0.4\cdot\arctan\left(\frac{D_f}{B}\right)$	1.0	1.0
	$\phi_f > 0$	$d_q - \frac{1-d_q}{N_q - 1}$	1.0	for $D_f \le B$: $1+2\cdot\tan\phi_f\cdot(1-\sin\phi_f)^2\cdot\frac{D_f}{B}$ for $D_f > B$: $1+2\cdot\tan\phi_f\cdot(1-\sin\phi_f)^2\cdot\arctan\left(\frac{D_f}{B}\right)$

Equations used for BC calculation

Table 29 Load inclination factors i_c, i_γ, i_q. (Vesić, 1975) (AASHTO Table 10.6.3.1.3a-5)

Factor	Friction Angle	Cohesion Term (i_c)	Unit Weight Term (i_γ)	Surcharge Term (i_q)
Load Inclination Factors i_c, i_γ, i_q	$\phi_f = 0$	$1 - \frac{n \cdot H}{c \cdot B \cdot L \cdot N_c}$	1.0	1.0
	$\phi_f > 0$	$i_q - \frac{1 - i_q}{N_q - 1}$	$\left[1 - \frac{H}{V + c \cdot B \cdot L \cdot \cot\phi_f}\right]^{(n+1)}$	$\left[1 - \frac{H}{V + c \cdot B \cdot L \cdot \cot\phi_f}\right]^{n}$

Soil parameters

Estimation based on SPT-N blow counts

Soil friction angle

Peck, Hanson and Thornburn (PHT) as modified by Kulhawy and Mayne (1990):

$$\phi_f \approx 54 - 27.6034 \cdot \exp\left(-0.014\left(N_1\right)_{60}\right)$$

- **N_{60} corrected using Liao and Whitman's correction (1996)**

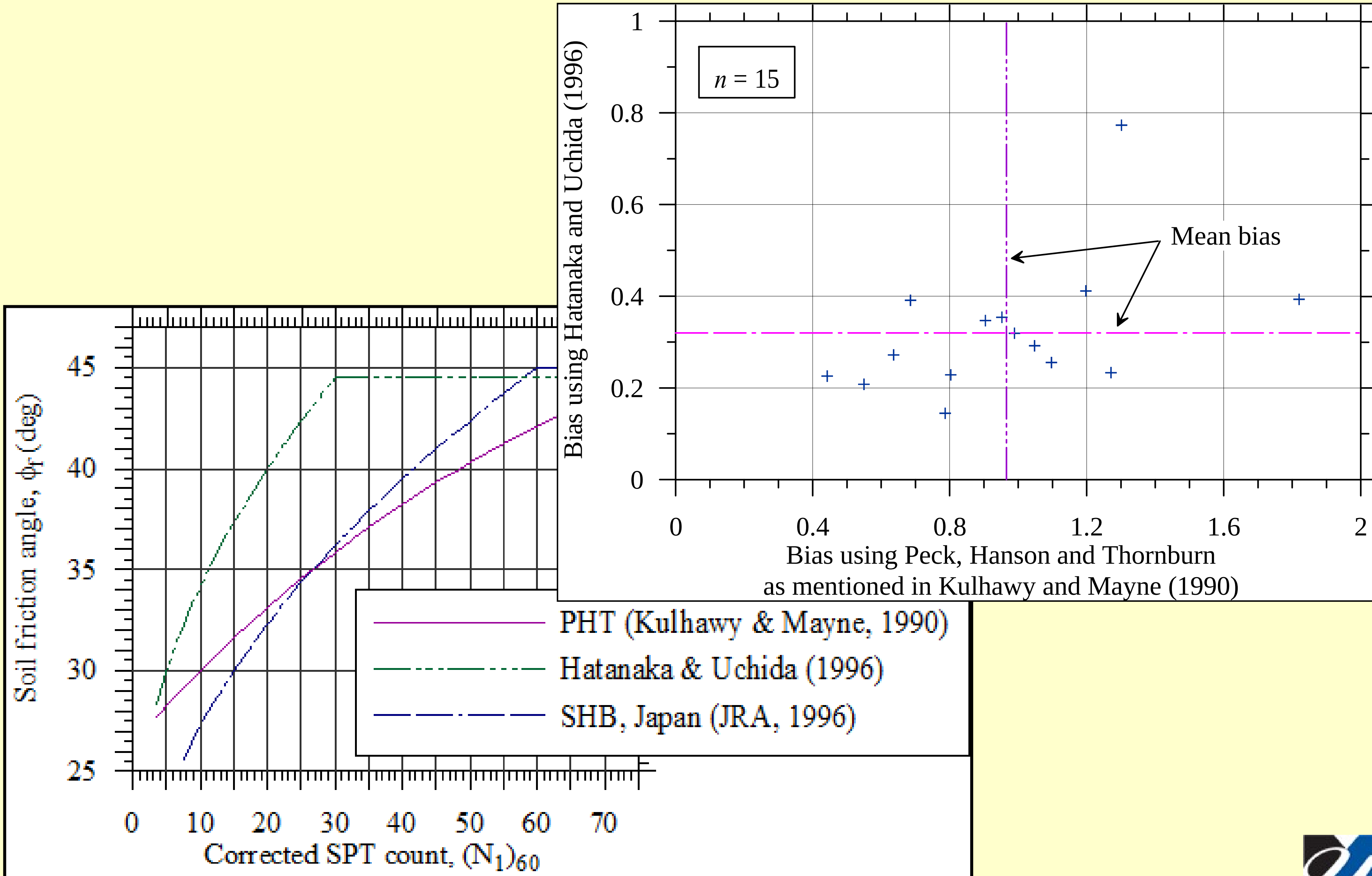
Soil unit weight

- **Paikowsky et at (1995):**

$$\gamma = 0.88(N_1)_{60} + 99 \quad \text{(pcf)} \qquad \text{for } \gamma \leq 146\text{pcf}$$

Estimation of ϕ_f from SPT-N

PHT and Hatanaka and Uchida – Comparison



Bias of Estimated BC

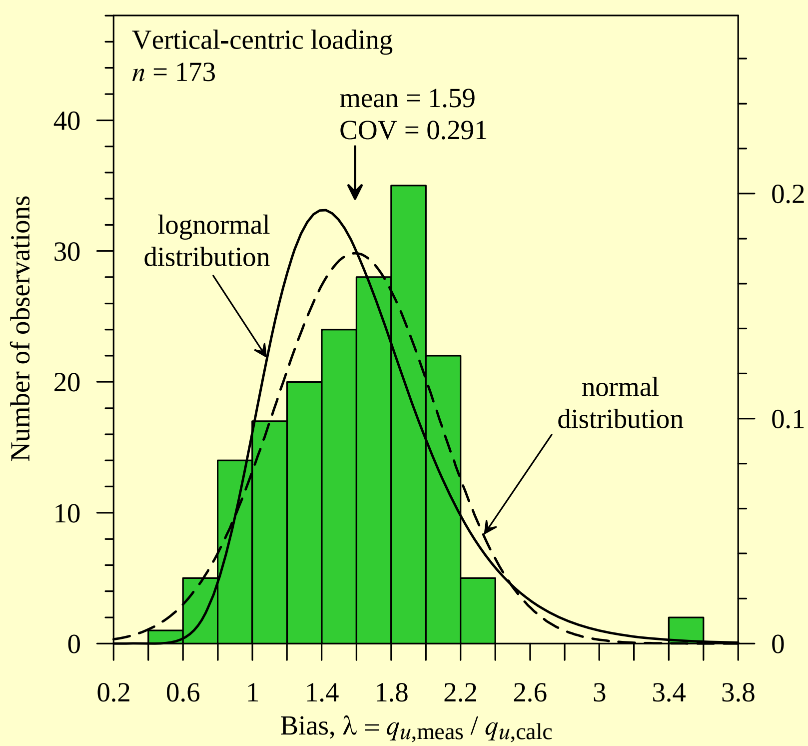
Cases with Vertical Centric Loading

- *Vertical Centric Loading*
n = 173; mean bias = 1.59, COV = 0.291
  - Natural soil conditions
(ϕ_f from SPT-N counts)
n = 14; no. of sites = 8
mean = 1.00
COV = 0.329
    - $B > 1.0$m
n = 6
no. of sites = 3
mean = 1.01
COV = 0.228
    - $0.1 < B \leq 1.0$m
n = 8
no. of sites = 7
mean = 0.99
COV = 0.407
  - Controlled soil conditions
($D_r \geq 35\%$)
n = 159; no. of sites = 7
mean = 1.64
COV = 0.267
    - $B \leq 0.1$m
n = 138
no. of sites = 5
mean = 1.67
COV = 0.245
    - $0.1 < B \leq 1.0$m
n = 21
no. of sites = 3
mean = 1.48
COV = 0.391

Figure 60 Summary of bias (measured over calculated BC) for vertical centric loading cases (Database I); 0.1m = 3.94in; 1m = 3.28ft.

Bias of Estimated BC

Cases with Vertical Centric Loading



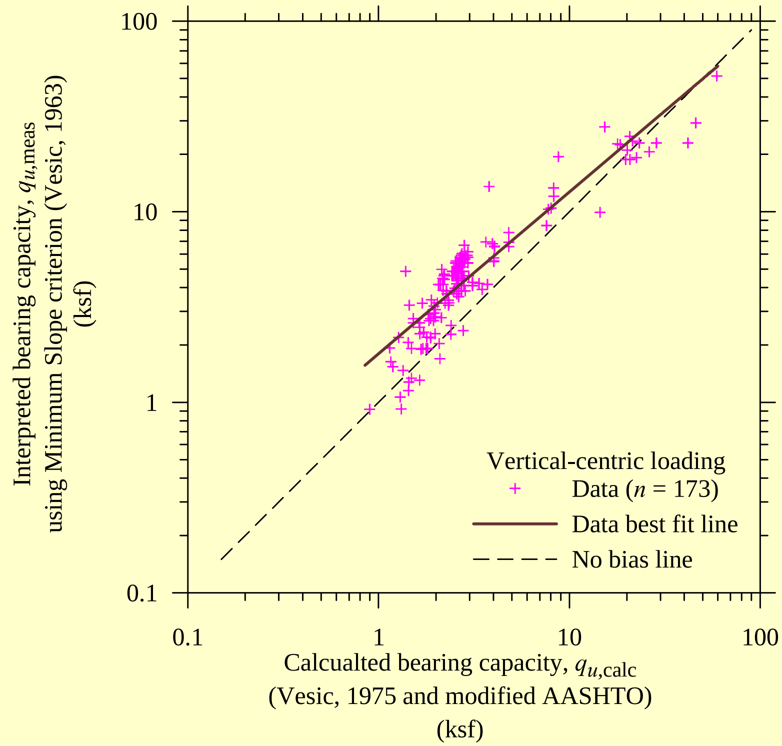
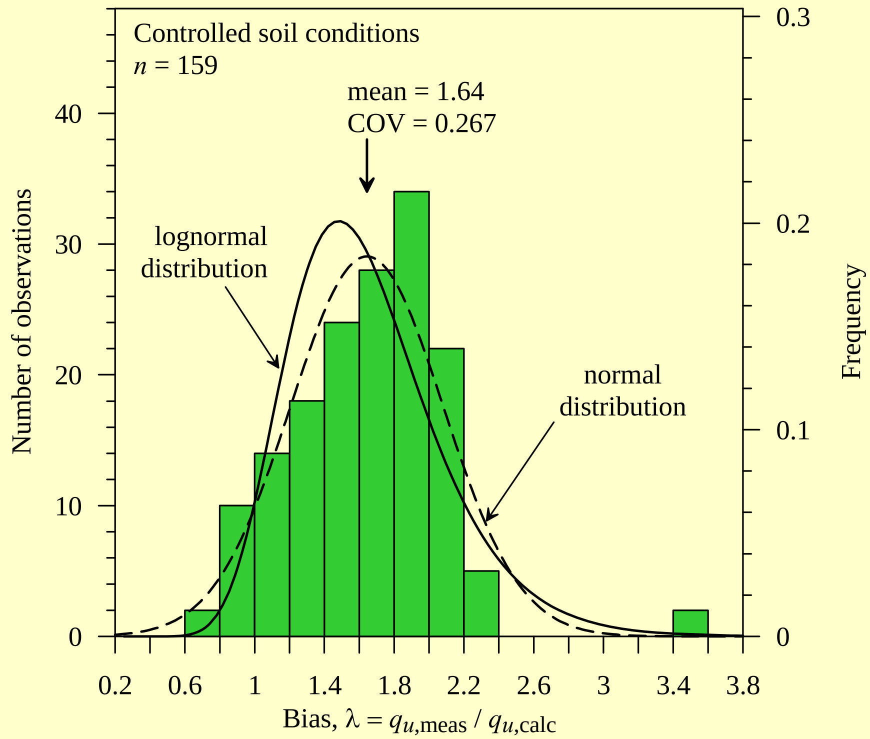


Figure 61. (a) Histogram and probability density functions of the bias and (b) relationship between measured and calculated bearing capacity for all cases of vertical centrically loaded shallow foundations.

Bias of Estimated BC

Cases with Vertical Centric Loading



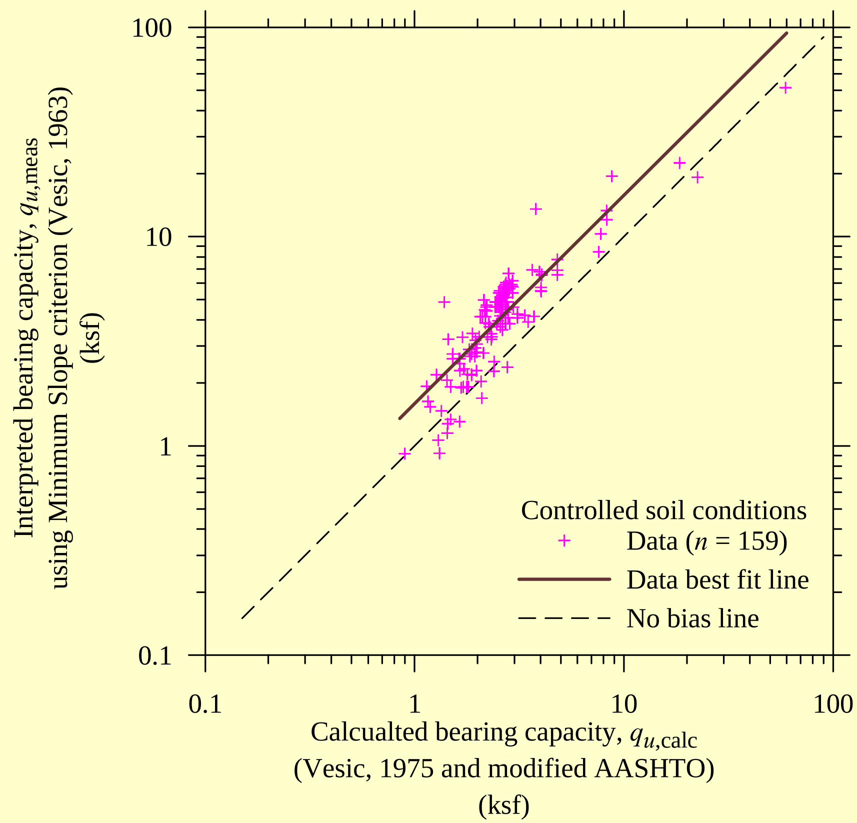


Figure 62. (a) Histogram and probability density functions of the bias and (b) relationship between measured and calculated bearing capacity for vertical centrically loaded shallow foundations on controlled soil conditions.

Bias of Estimated BC

Cases with Vertical Centric Loading

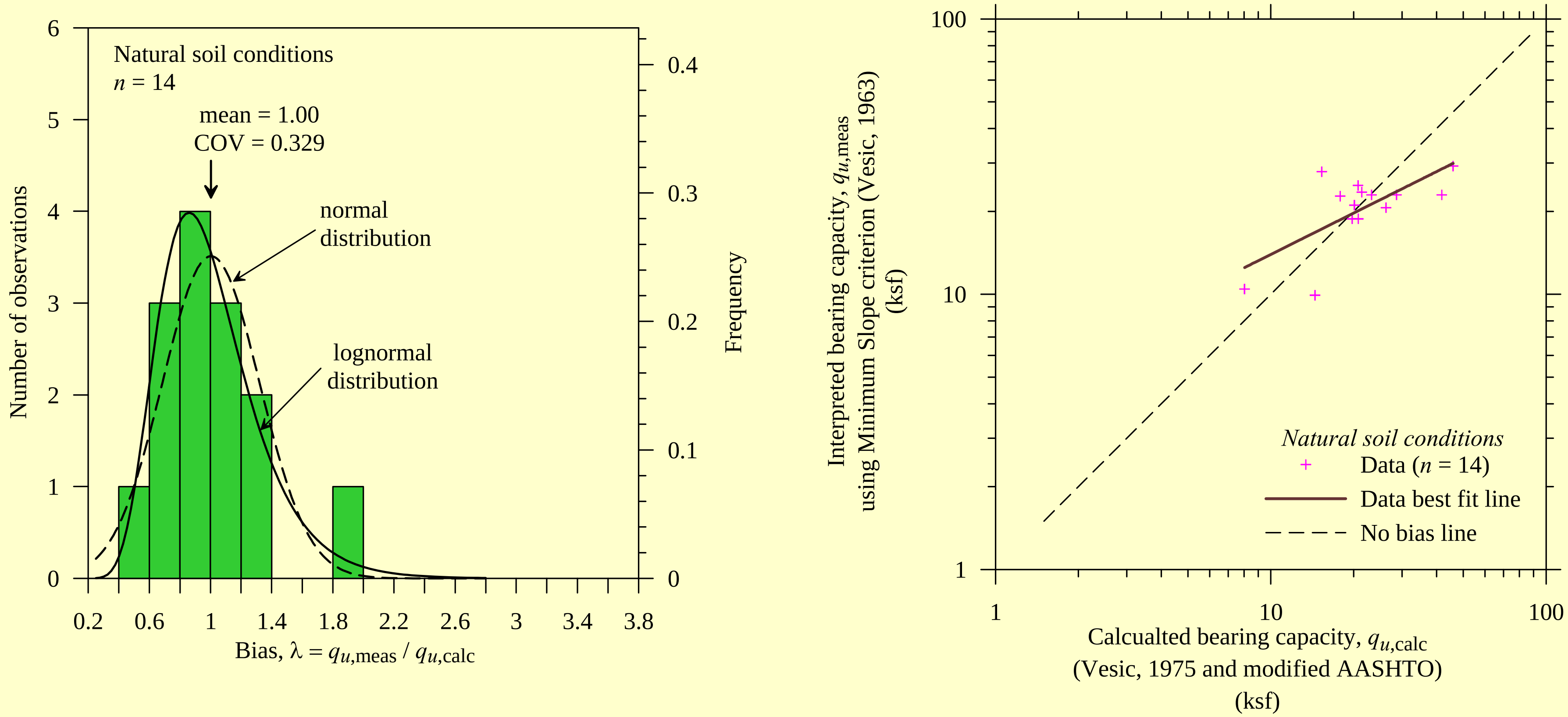


Figure 63. (a) Histogram and probability density functions of the bias and (b) relationship between measured and calculated bearing capacity for vertical centrically loaded shallow foundations on natural soil conditions.

Bias versus Footing Width

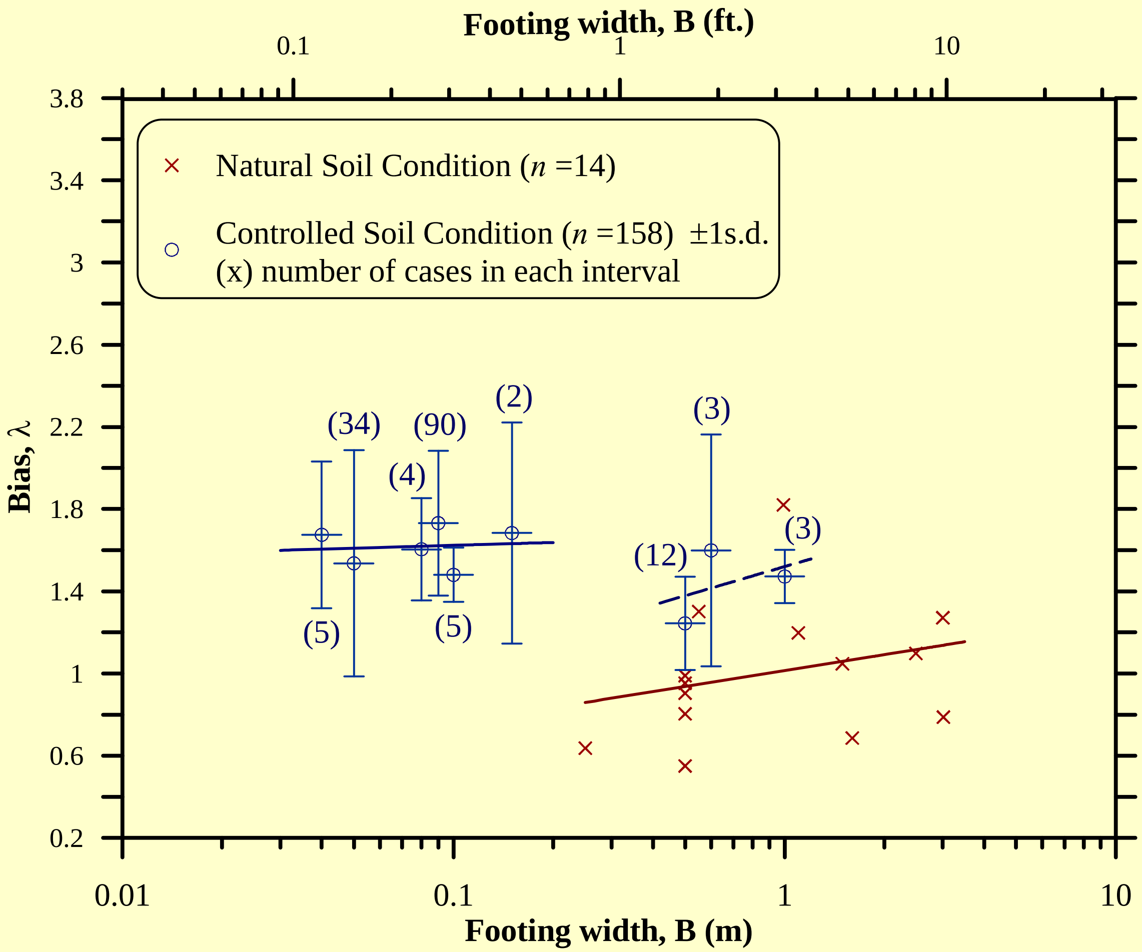


Figure 99. Variation of the bias in bearing resistance versus footing size for cases under vertical-centric loadings: controlled and natural soil conditions.

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Bias versus Footing Width

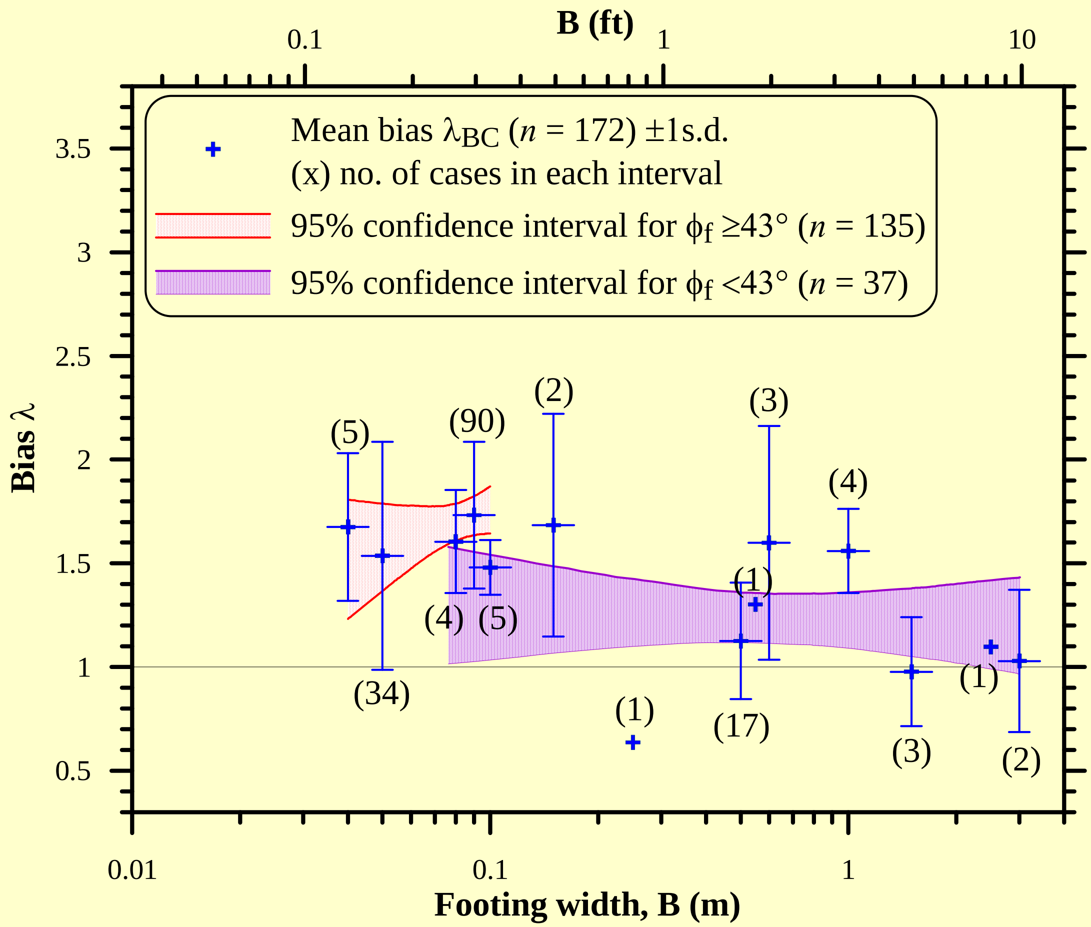
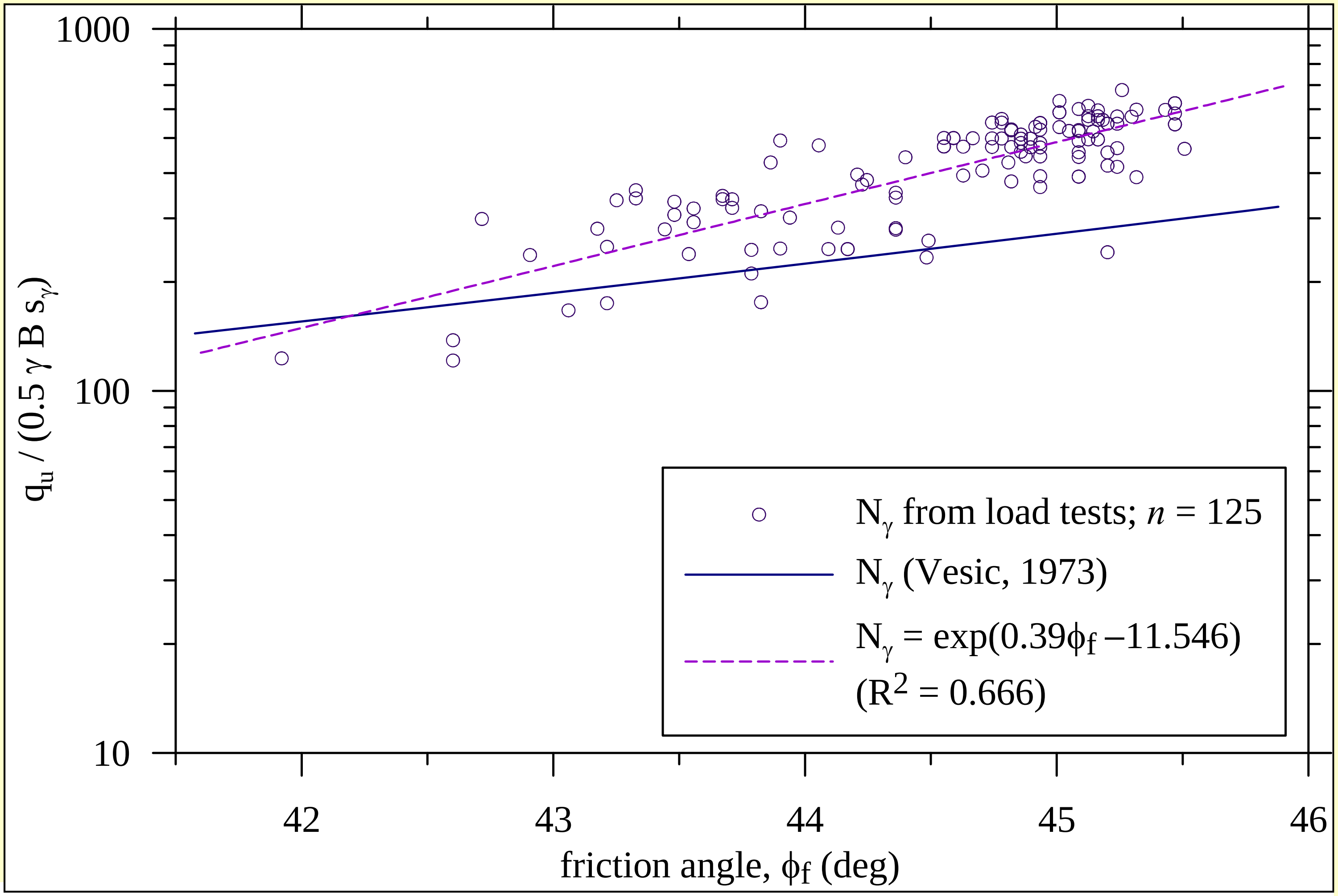


Figure 100. Variation of the bias in bearing resistance versus footing size for cases under vertical-centric loadings: $\phi_f \geq 43°$ and $\phi_f < 43°$.

Uncertainty in N_γ



Comparison of bearing capacity factor calculated based on test results; $N_\gamma = q_u / (0.5\gamma B s_\gamma)$ from 125 tests carried out in controlled soil conditions (tests by Perau, 1995) and N_γ proposed by Vesic (1973) in the range of soil friction angle of 42° and 46°

Uncertainty in N_γ

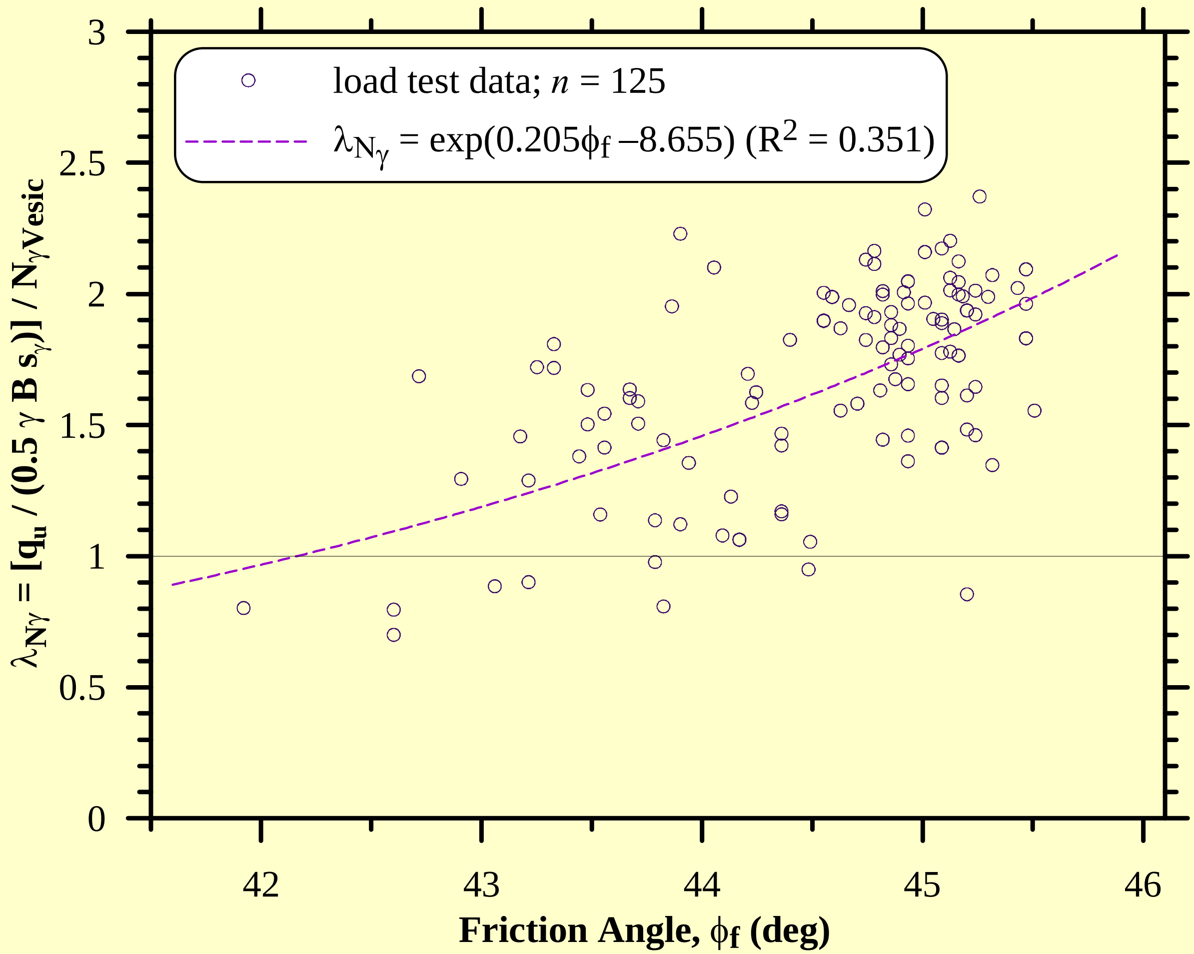


Figure 93. The ratio ($\lambda_{N\gamma}$) between the back-calculated B.C. factor N_γ based on experimental data to that proposed by Vesić versus soil friction angle.

Uncertainty in N_γ

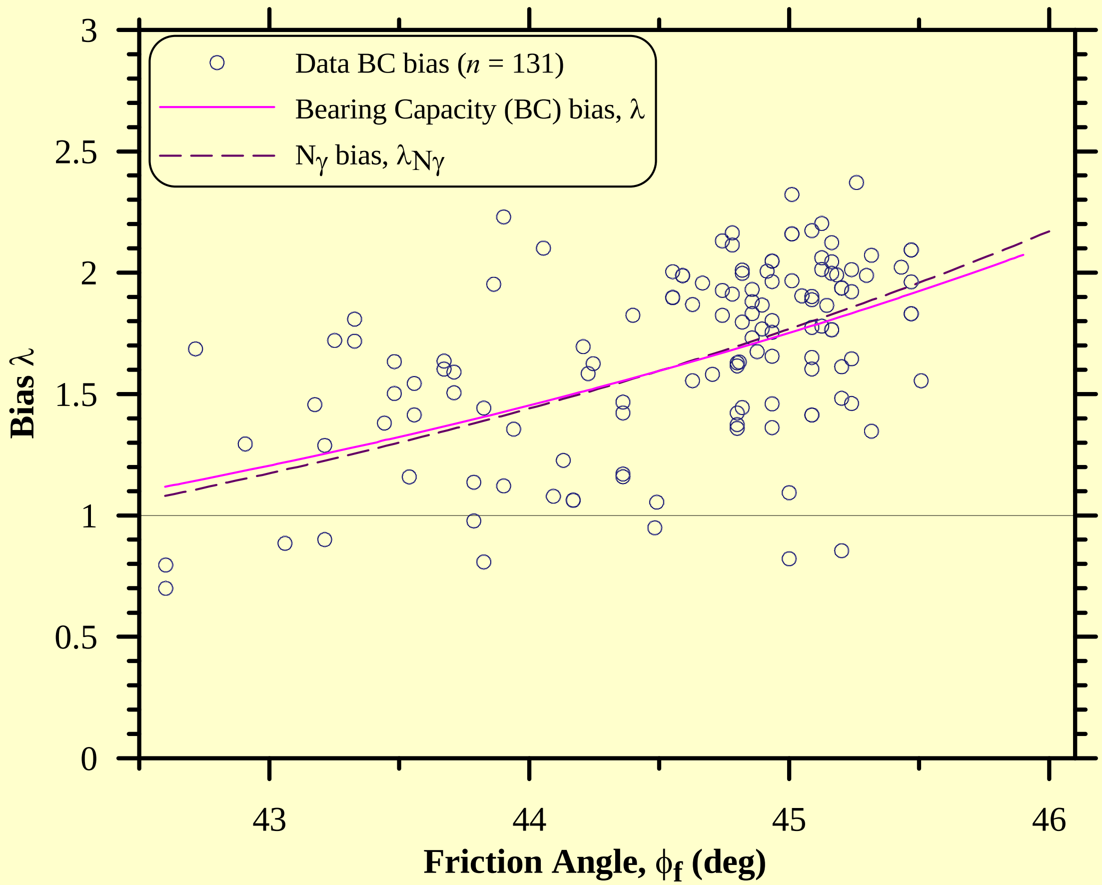


Figure 94. The ratio between measured and calculated bearing capacity (bias λ) compared to the bias in the B.C. factor N_γ ($\lambda_{N\gamma}$) versus the soil's friction angle for footings under vertical-centric loadings.

Uncertainty in B.C.

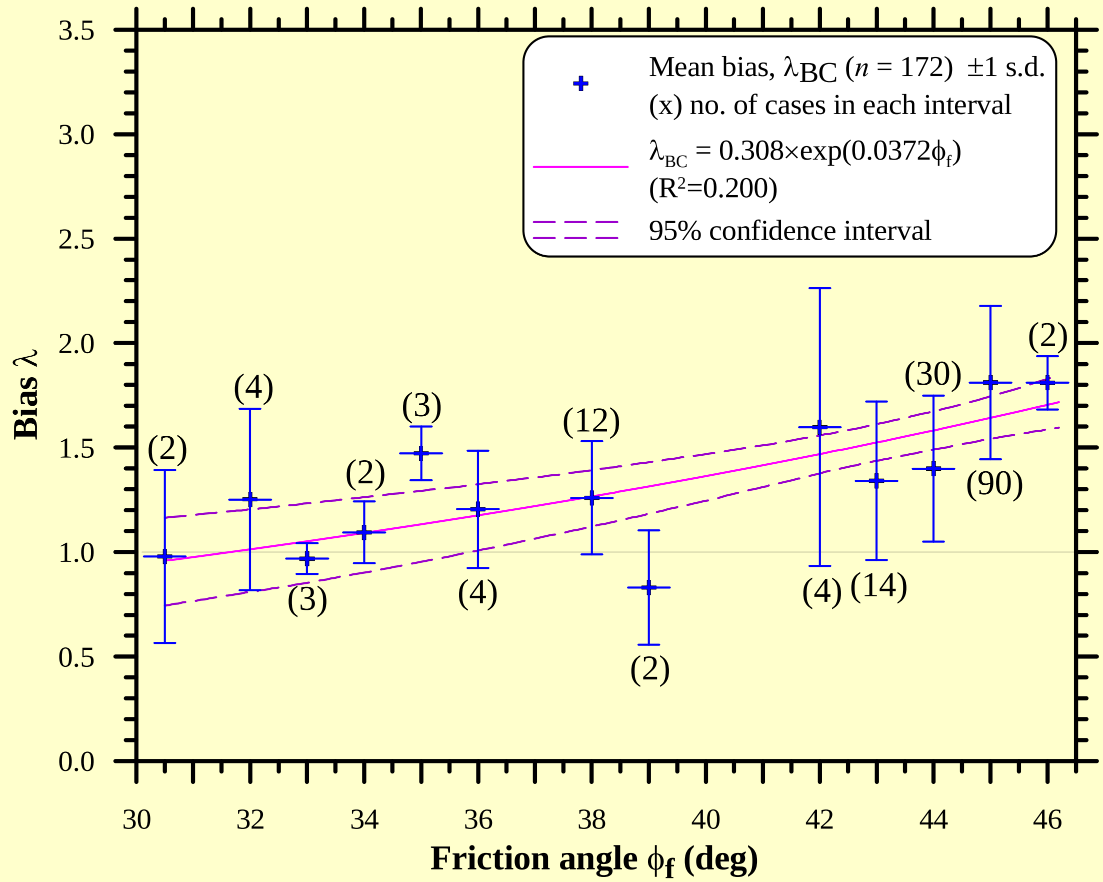


Figure 103. Bearing resistance bias vs. average soil friction angle (taken $\phi_f \pm 0.5°$) including 95% confidence interval for all cases under vertical-centric loading.

Uncertainty in B.C.

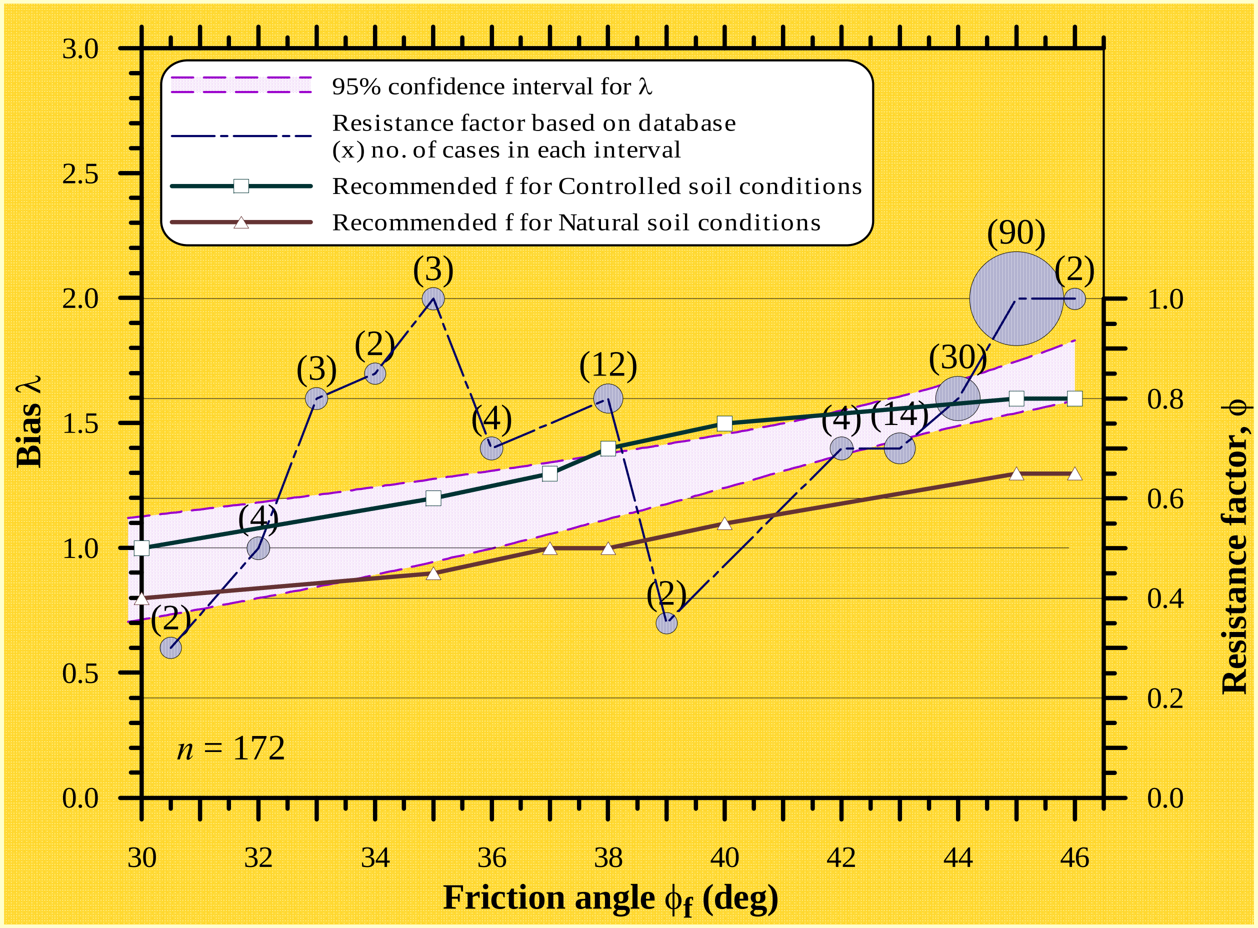


Figure 104. Recommended resistance factors for soil friction angles (taken $\phi_f \pm 0.5°$) between 30° and 46°, with comparisons to 95% confidence interval and resistance factors obtained for the cases in the database; the bubble size represents the number of data cases in each subset.

Uncertainty in B.C.

Table 58 Resistance factors for vertical-centric loading cases based on the bias vs. ϕ_f best-fit line of equation (121) and the COV of natural vs. controlled soil conditions

Soil friction angle ϕ_f (deg)	Mean bias λ (Equation 121)	Resistance factor ϕ (β_T = 3) Soil Condition: Natural (COV_λ = 0.35)		Resistance factor ϕ (β_T = 3) Soil Condition: Controlled (COV_λ = 0.25)	
		MCS	Reco	MCS	Reco
30	0.94	0.403	0.40	0.542	0.50
35	1.13	0.485	0.45	0.652	0.60
37	1.22	0.524	0.50	0.703	0.70
38	1.27	0.545	0.50	0.732	0.70
40	1.36	0.584	0.55	0.784	0.75
≥45	1.64	0.704	0.65	0.946	0.80

5. Calibration of Resistance Factors

Vertical Load Distributions

Using the same distributions used for strength limit state for piles (NCHRP 507) and service limit state of foundations (NCHRP 12-66):

Based on Nowak (1999) NCHRP 368

Dead Load $\gamma_D = 1.25$ $\lambda_{QD} = 1.05$ $COV_{QD} = 0.1$
(as recommended by Nowak)

Live Load $\gamma_L = 1.75$ $\lambda_{QL} = 1.15$ $COV_{QL} = 0.2$
Table F1 by Nowak $\lambda_{QL} = 1.1$ **to** 1.2, $COV_{QL} = 0.18$
(Selected in consultation with Billal Ayyub)

Dead to Live Load Ratio = 2.0 (see discussion in NCHRP 507)

Recommended Resistance Factors for Vertical-Centric Loading

Table 59 Recommended resistance factors for Vertical-Centric loading cases

Soil friction angle ϕ_f (deg)	Recommended resistance factor ϕ ($\beta_T = 3$) Soil Conditions	
	Natural	Controlled
30 – 34	0.40	0.50
35 – 36	0.45	0.60
37 – 39	0.50	0.70
40 – 44	0.55	0.75
≥ 45	0.65	0.80

Final Resistance Factors – Controlled Conditions

Table 66 Recommended resistance factors for shallow foundations on granular soils placed under controlled conditions

<table>
<tr><th rowspan="3">Soil friction angle ϕ_f</th><th colspan="4">Loading conditions</th></tr>
<tr><th rowspan="2">Vertical-centric or -eccentric</th><th rowspan="2">Inclined-centric</th><th colspan="2">Inclined-eccentric</th></tr>
<tr><th>Positive</th><th>Negative</th></tr>
<tr><td>30° – 34°</td><td>0.50</td><td rowspan="2">0.40</td><td rowspan="2">0.40</td><td rowspan="2">0.70</td></tr>
<tr><td>35° – 36°</td><td>0.60</td></tr>
<tr><td>37° – 39°</td><td>0.70</td><td>0.45</td><td>0.45</td><td>0.75</td></tr>
<tr><td>40° – 44°</td><td>0.75</td><td>0.50</td><td rowspan="2">0.50</td><td rowspan="2">0.80</td></tr>
<tr><td>≥ 45°</td><td>0.80</td><td>0.55</td></tr>
</table>

Notes:

1) ϕ_f determined by laboratory testing
2) compacted controlled fill or improved ground are assumed to extend below the base of the footing to a distance to at least two (2.0) times the width of the foundation (B). If the fill is less than 2B thick, but overlays a material equal or better in strength than the fill itself, then the recommendation stands. If not, then the strength of the weaker material within a distance of 2B below the footing; prevails.
3) The resistance factors were evaluated for a target reliability β_T = 3.0.

Final Resistance Factors – Natural Conditions

Table 67 Recommended resistance factors for shallow foundations on natural deposited granular soil conditions

<table>
<tr><th rowspan="3">Soil friction angle ϕ_f</th><th colspan="4">Loading conditions</th></tr>
<tr><th rowspan="2">Vertical-centric or -eccentric</th><th rowspan="2">Inclined-centric</th><th colspan="2">Inclined-eccentric</th></tr>
<tr><th>Positive</th><th>Negative</th></tr>
<tr><td>30° – 34°</td><td>0.40</td><td rowspan="3">0.40</td><td rowspan="2">0.35</td><td>0.65</td></tr>
<tr><td>35° – 36°</td><td>0.45</td><td rowspan="2">0.70</td></tr>
<tr><td>37° – 39°</td><td>0.50</td><td rowspan="2">0.40</td></tr>
<tr><td>40° – 44°</td><td>0.55</td><td>0.45</td><td rowspan="2">0.75</td></tr>
<tr><td>≥ 45°</td><td>0.65</td><td>0.50</td><td>0.45</td></tr>
</table>

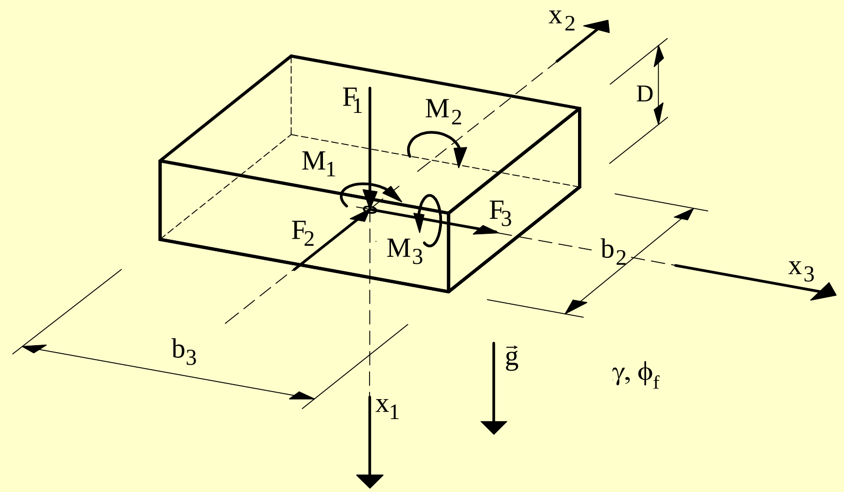
Notes:

1) ϕ_f determined from Standard Penetration Test results
2) granular material is assumed to extend below the base of the footing at least two (2.0) times the width of the foundation.
3) The resistance factors were evaluated for a target reliability β_T = 3.0

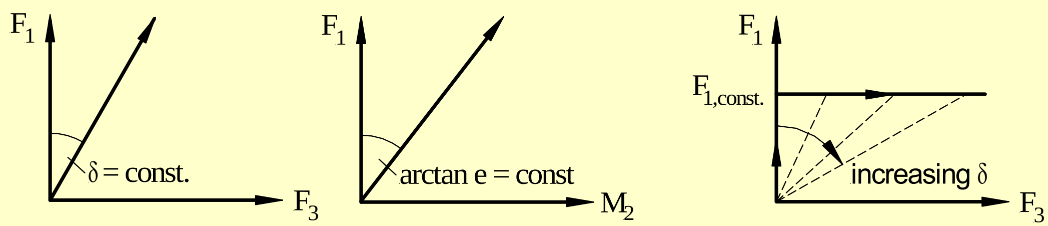
Intermediate Conclusions and Summary

- **It was found that for the footings of larger sizes (B>3m (9.9ft)), the load tests were not carried out to the failure load**
- **Biases for the tests in Natural Soil Condition and Controlled Soil Conditions were analyzed separately**
- **For the footing sizes in similar ranges (0.1m < B $\leq$ 1.0m), the scatter of bias was larger for footings on/in natural soil conditions**
- **The majority of the relevant data refers to small size foundations (B $\leq$ 3.3ft (1.0m)) on controlled compacted material. Many of the highway shallow foundations on soils are built on compacted materials and hence, the statistical data of the uncertainty can be used for that purpose**
- **There appears to be a trend of increase in bias with the footing size within the range of footing sizes available for testing (which seems to conform with the observation made by Vesic (1969))**

ULS of Inclined Loading



(a) Loading convention



(b) Radial load path

(c) Step-like load path

Figure 64. Loading convention and load paths used during tests.

ULS of Inclined Loading

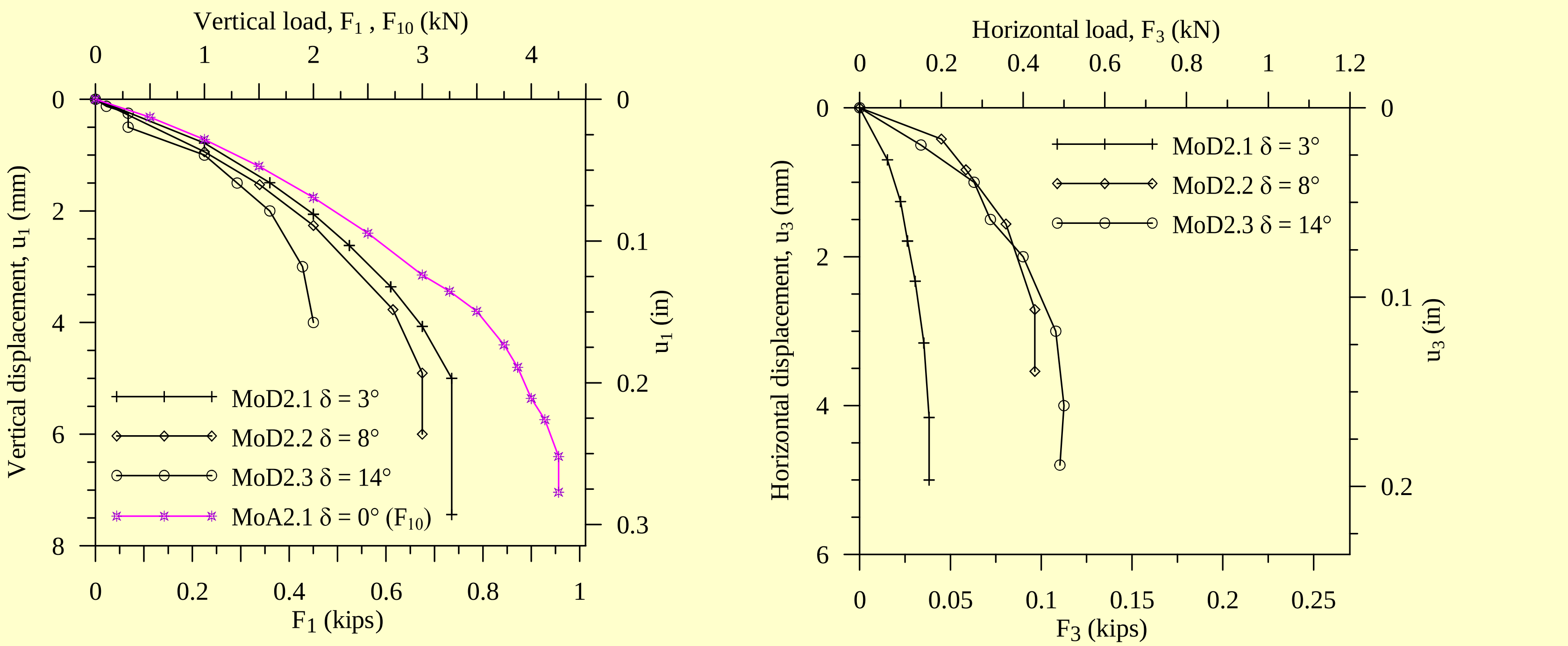


Figure 65. Load–displacements curves for model tests conducted by Montrasio (1994) with varying load inclination: (a) vertical load vs. vertical displacement and (b) horizontal load vs. horizontal displacement.

Bias of Estimated BC

Cases with Vertical-Eccentric Loading (using B′)

Table 32 Summary of the statistics for biases of the test results for vertical-eccentric loading when using effective foundation width B′

Tests	No. of cases	Minimum slope criterion			Two slope criterion		
		Mean	Std. Dev.	COV	Mean	Std. Dev.	COV
DEGEBO – radial load path	17 (15)[1]	2.22	0.754	0.340	2.04	0.668	0.328
Montrasio/Gottardi – radial load path	14	1.71	0.399	0.234	1.52	0.478	0.313
Perau – radial load path	12	1.43	0.337	0.263	1.19	0.470	0.396
All cases	43 (41)[1]	1.83	0.644	0.351	1.61	0.645	0.400

Radial Load Path – Gradual increase of loads keeping the eccentricity constant

Bias of Estimated BC

Cases with Vertical-Eccentric Loading (using B)

Table 33 Summary of the statistics for biases of the test results for vertical-eccentric loading when using foundation width B

Tests	No. of cases	Minimum slope criterion			Two slope criterion		
		Mean	Std. Dev.	COV	Mean	Std. Dev.	COV
DEGEBO – radial load path	17 (15)[1]	1.30	0.464	0.358	1.20	0.425	0.355
Montrasio/Gottardi – radial load path	14	0.97	0.369	0.380	0.86	0.339	0.396
Perau – radial load path	12	0.79	0.302	0.383	0.64	0.296	0.465
All cases	43 (41)[1]	1.05	0.441	0.420	0.92	0.423	0.461

Radial Load Path – Gradual increase of loads keeping the eccentricity constant

Bias of Estimated BC

Cases with Vertical-Eccentric Loading (using B′)

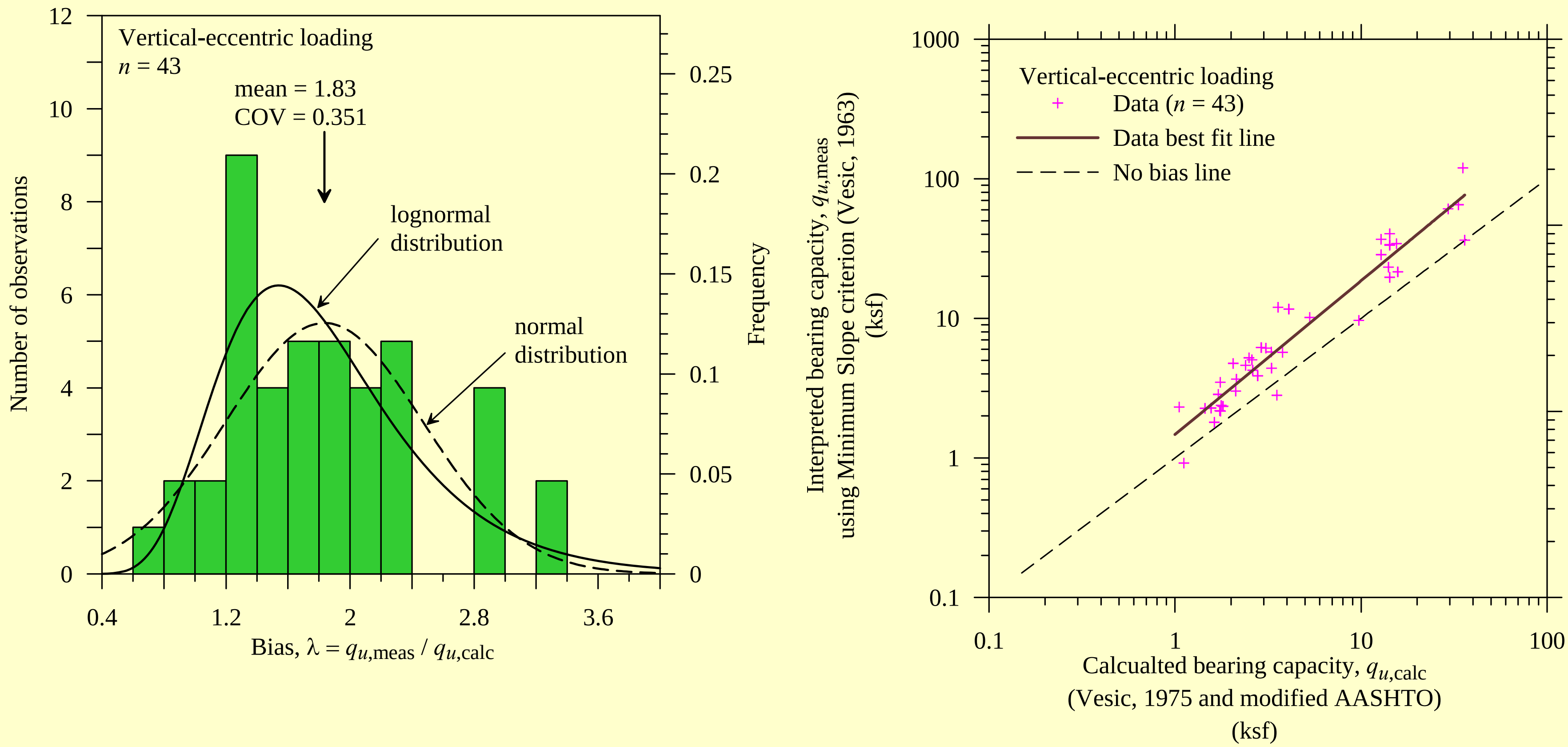


Figure 66. (a) Histogram and probability density functions of the bias and (b) relationship between measured and calculated bearing capacity for all cases of vertical eccentrically loaded shallow foundations.

Bias of Estimated BC

Cases with Vertical-Eccentric Loading (using B)

Table 33 Summary of the statistics for biases of the test results for vertical-eccentric loading when using foundation width B

Tests	No. of cases	Minimum slope criterion			Two slope criterion		
		Mean	Std. Dev.	COV	Mean	Std. Dev.	COV
DEGEBO – radial load path	17 (15)[1]	1.30	0.464	0.358	1.20	0.425	0.355
Montrasio/Gottardi – radial load path	14	0.97	0.369	0.380	0.86	0.339	0.396
Perau – radial load path	12	0.79	0.302	0.383	0.64	0.296	0.465
All cases	43 (41)[1]	1.05	0.441	0.420	0.92	0.423	0.461

[1] number of cases for Two slope criterion

Bias of Estimated BC

Cases with Vertical-Eccentric Loading

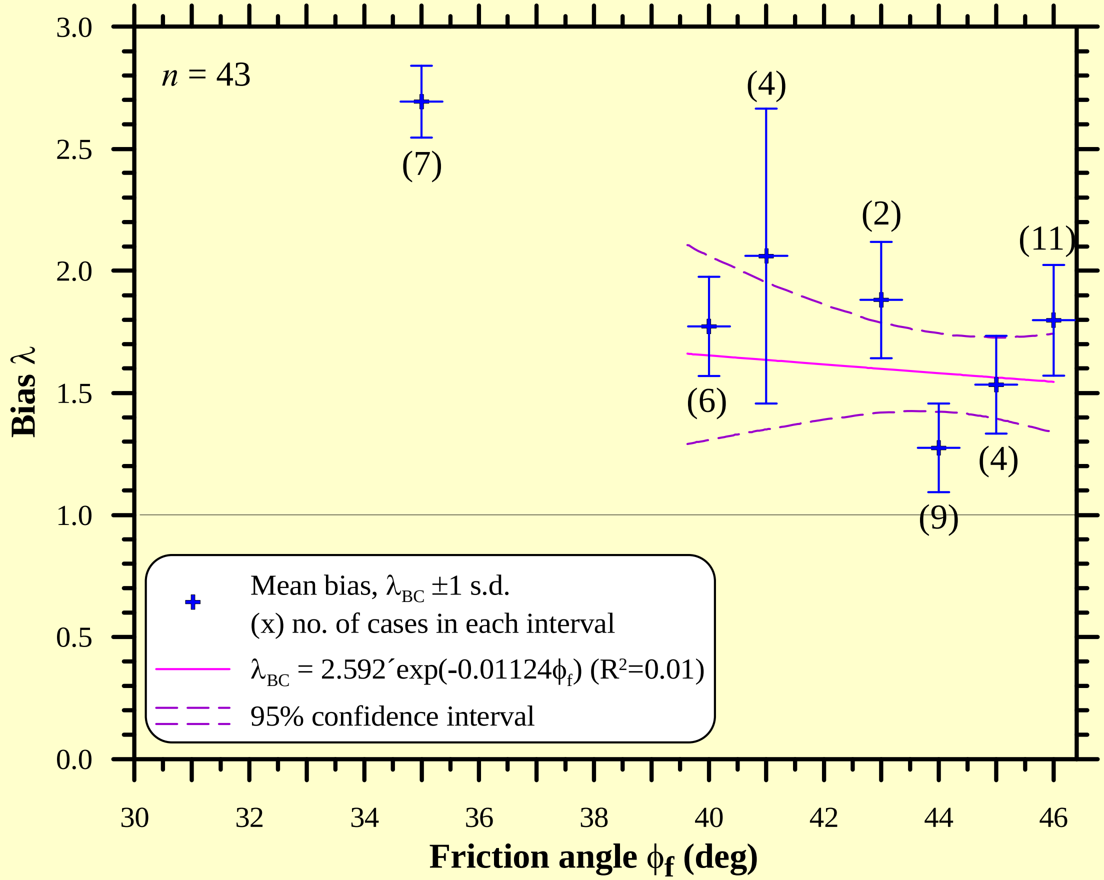


Figure 105. Bearing resistance bias versus soil friction angle for cases under vertical-eccentric loadings; seven cases for ϕ_f = 35° (all from a single site) have been ignored for obtaining the best fit line.

Bias of Estimated BC

Cases with Vertical-Eccentric Loading

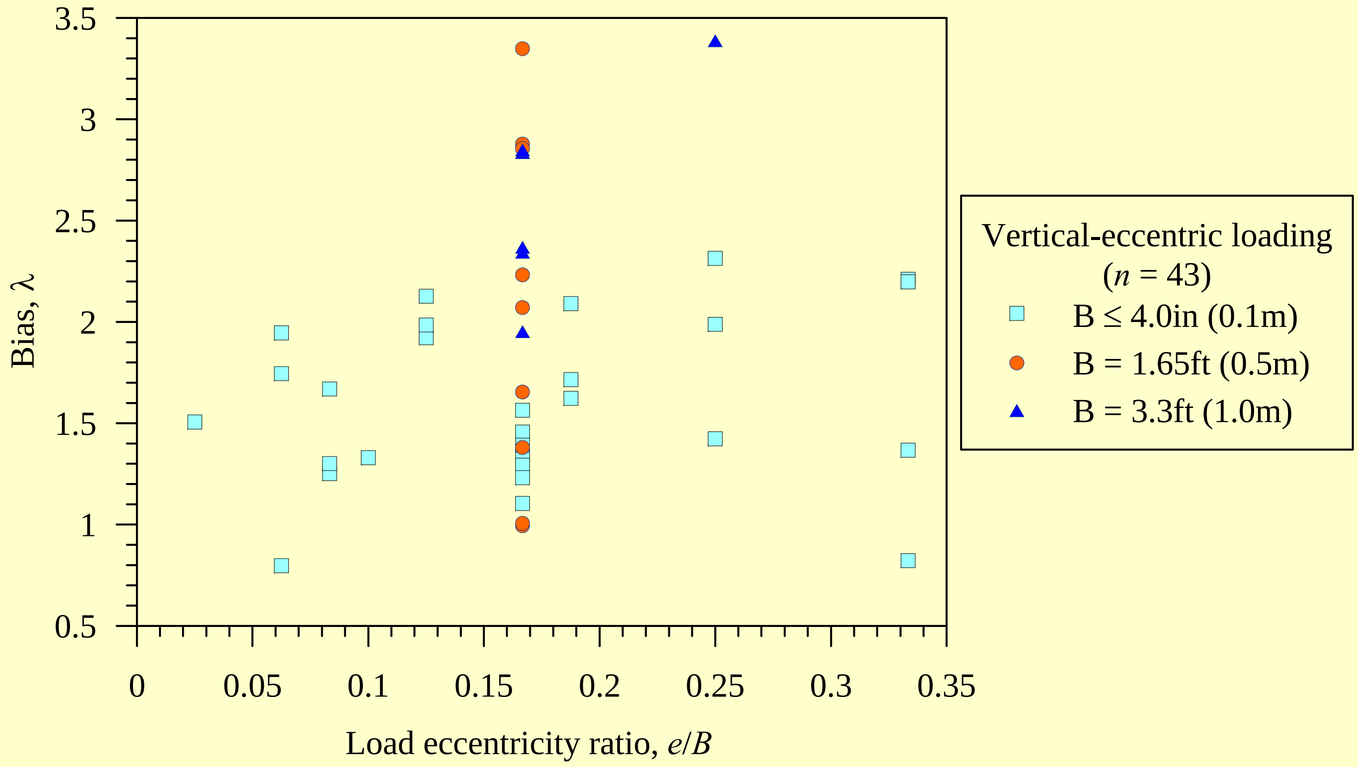


Figure 106. Bearing resistance bias vs. load eccentricity ratio e/B for vertical-eccentric loading.

Bias of Estimated BC

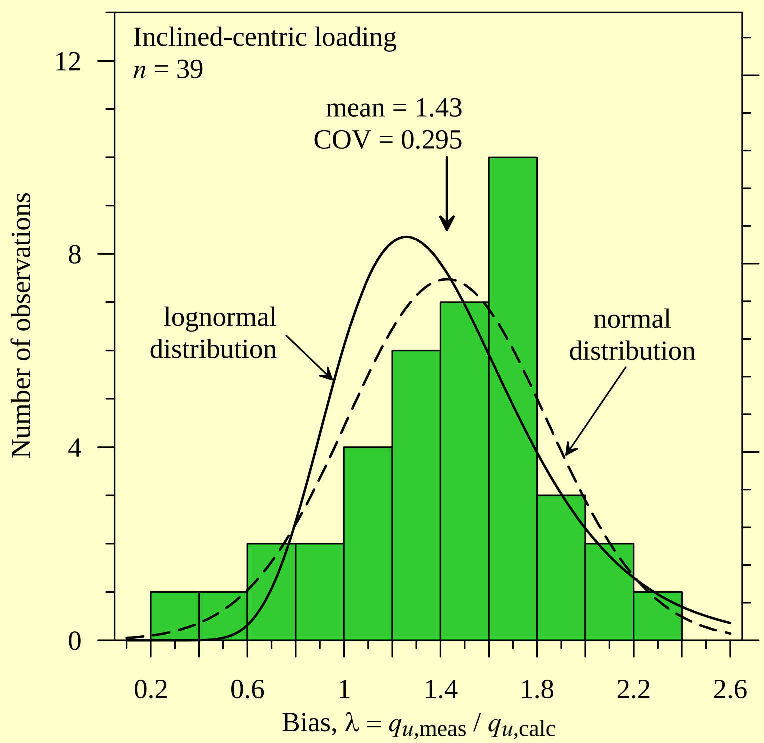
Cases with Inclined-Centric Loading (using B)

Tests	No. of cases	Minimum slope criterion			Two slope criterion		
		Mean	Std. Dev.	COV	Mean	Std. Dev.	COV
DEGEBO/ Montrasio/Gottardi – radial load path	26 (24)[1]	1.56	0.346	0.222	1.35	0.452	0.334
Perau/Gottardi – step-like load path	13	1.17	0.537	0.459	1.17	0.537	0.459
All cases	39 (37)[1]	1.43	0.422	0.295	1.29	0.455	0.353

[1] number of cases for Two slope criterion

Bias of Estimated BC

Cases with Inclined-Centric Loading



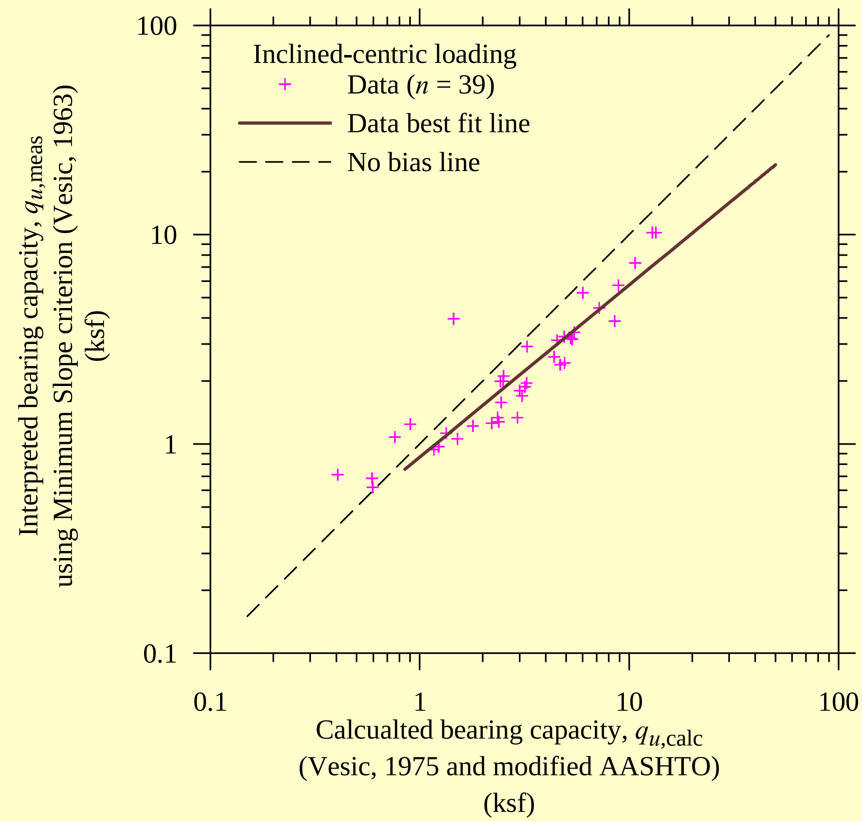


Figure 67. (a) Histogram and probability density functions of the bias and (b) relationship between measured and calculated bearing capacity for all cases of inclined centric loaded shallow foundations.

Bias of Estimated BC

Cases with Inclined-Eccentric Loading (using B′)

Table 35 Summary of the statistics for biases of the test results for inclined-eccentric loading when using effective foundation width B′

Tests		No. of cases	Minimum slope criterion			Two slope criterion		
			Mean	**Std. Dev.**	**COV**	**Mean**	**Std. Dev.**	**COV**
DEGEBO/Gottardi – radial load path		8	2.06	0.813	0.394	1.78	0.552	0. 310
Step-like load path	Montrasio/ Gottardi	6	2.13	0.496	0.234	2.12	0.495	0.233
	Perau – positive eccentricity	8	2.16	1.092	0.506	2.15	1.073	0.500
	Perau – negative eccentricity	7	3.43	1.792	0.523	3.39	1.739	0.513
	All step-like load cases	21	2.57	1.352	0.526	2.56	1.319	0.516
All cases		29	2.43	1.234	0.508	2.34	1.201	0.513

Bias of Estimated BC

Cases with Inclined-Eccentric Loading

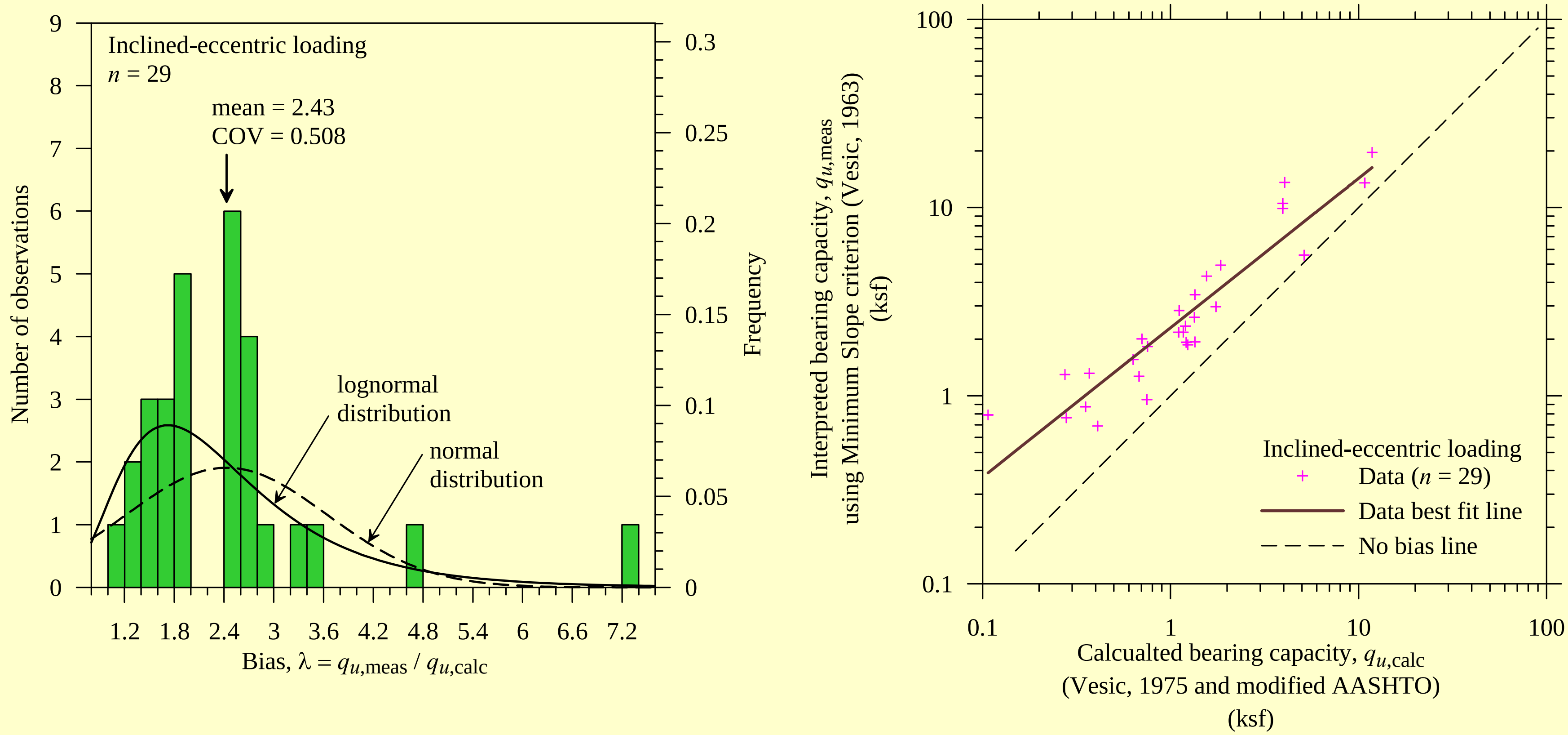


Figure 68. (a) Histogram and probability density functions of the bias and (b) relationship between measured and calculated bearing capacity for all cases of inclined eccentrically loaded shallow foundations.

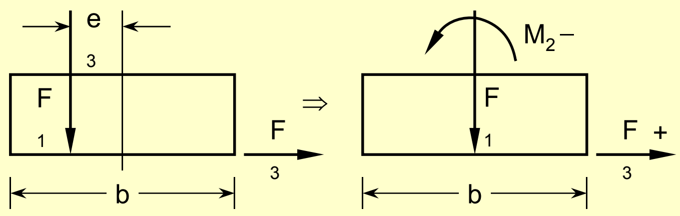
Bias of Estimated BC

Cases with Inclined-Eccentric Loading (using B′)

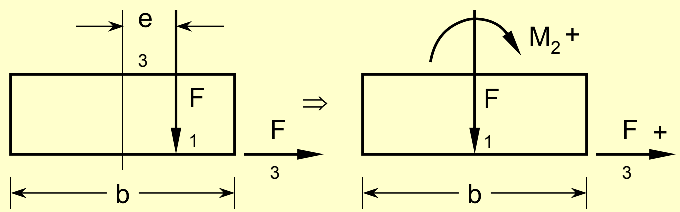
Table 36 Summary of the statistics for biases of the test results for inclined-eccentric loading when using foundation width B

Tests		No. of cases	Minimum slope criterion			Two slope criterion		
			Mean	Std. Dev.	COV	Mean	Std. Dev.	COV
DEGEBO/Gottardi – radial load path		8	1.07	0.448	0.417	0.94	0.365	0. 387
Step-like load path	Montrasio/ Gottardi	6	1.18	0.126	0.106	1.18	0.125	0.106
	Perau – positive eccentricity	8	0.70	0.136	0.194	0.70	0.135	0.194
	Perau – negative eccentricity	7	1.09	0.208	0.191	1.08	0.208	0.193
	All step-like load cases	21	0.97	0.267	0.276	0.96	0.267	0.277
All cases		29	1.00	0.322	0.323	0.96	0.290	0.303

Loading Directions for Inclined-Eccentric Loadings

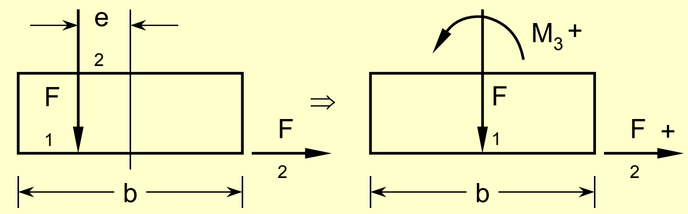


Moment acting in direction opposite to the lateral loading – negative eccentricity

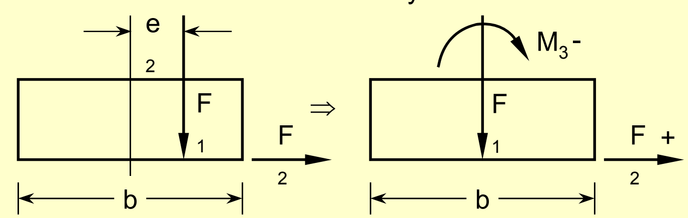


Moment acting in the same direction as the lateral loading – positive eccentricity

(a) along footing width



Moment acting in direction opposite to the lateral loading – negative eccentricity



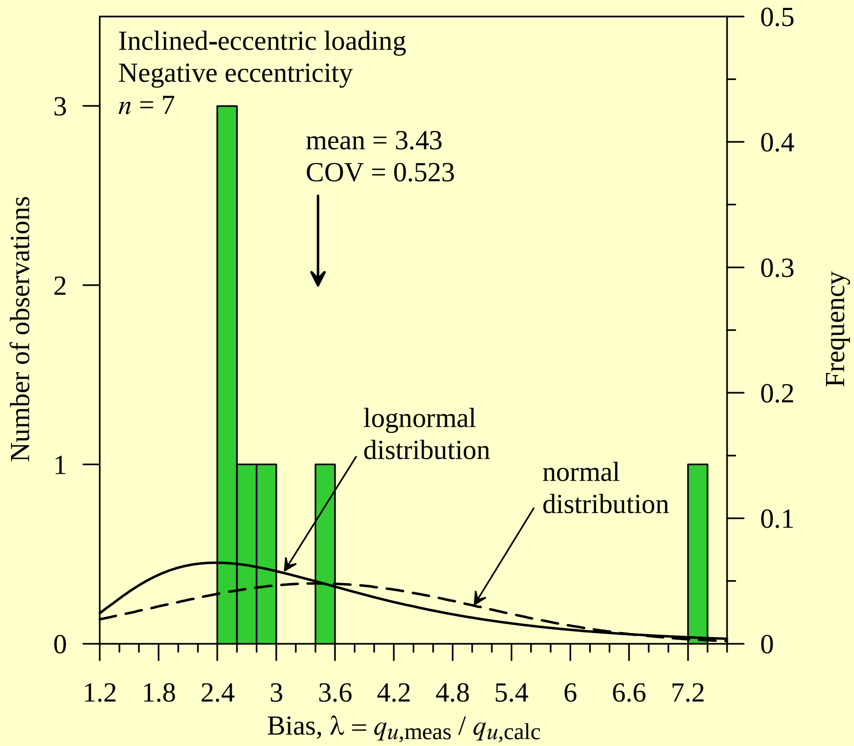
Moment acting in the same direction as the lateral loading – positive eccentricity

(b) along footing length

Figure 69. Loading directions for the case of inclined-eccentric loadings: (a) along footing width and (b) along footing length

Bias of Estimated BC

Cases with Inclined-Eccentric Loading



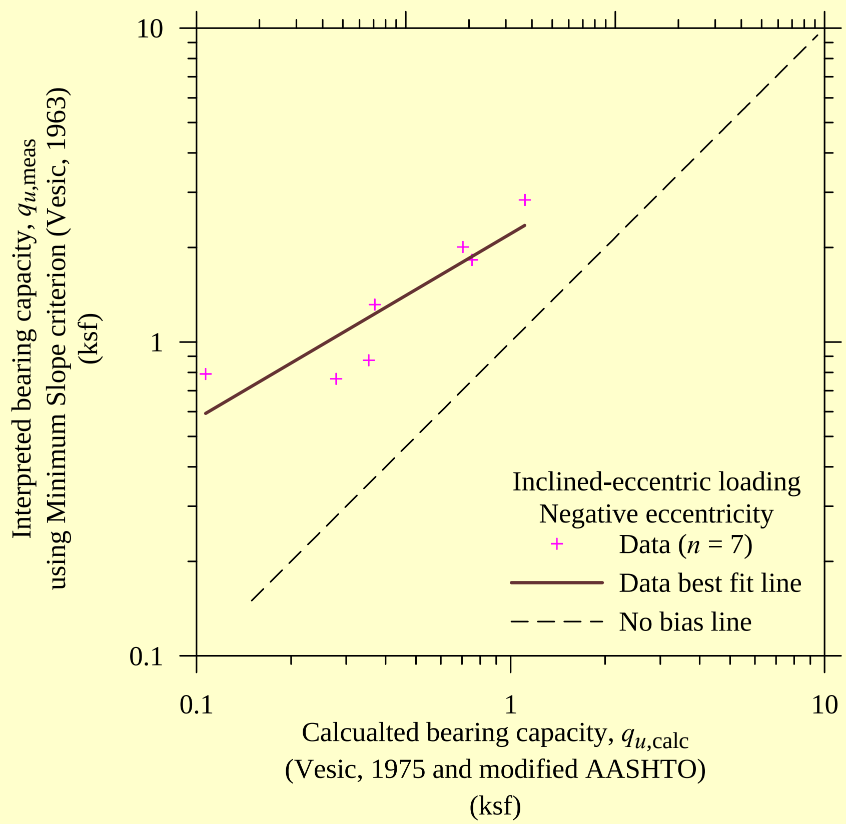


Figure 71. (a) Histogram and probability density functions of the bias and (b) relationship between measured and calculated bearing capacity for all cases of inclined eccentrically loaded shallow foundations under negative eccentricity.

5. Calibration of Resistance Factors

Vertical Load Distributions

Using the same distributions used for strength limit state for piles (NCHRP 507) and service limit state of foundations (NCHRP 12-66):

Based on Nowak (1999) NCHRP 368

Dead Load $\gamma_D = 1.25$ $\lambda_{QD} = 1.05$ $COV_{QD} = 0.1$
(as recommended by Nowak)

Live Load $\gamma_L = 1.75$ $\lambda_{QL} = 1.15$ $COV_{QL} = 0.2$
Table F1 by Nowak $\lambda_{QL} = 1.1$ to 1.2, $COV_{QL} = 0.18$
(Selected in consultation with Billal Ayyub)

Dead to Live Load Ratio = 2.0 (see discussion in NCHRP 507)

Final Resistance Factors – Natural Conditions

Table 67 Recommended resistance factors for shallow foundations on <u>natural deposited</u> granular soil conditions

<table>
<tr><th rowspan="3">Soil friction angle ϕ_f</th><th colspan="4">Loading conditions</th></tr>
<tr><th rowspan="2">Vertical-centric or -eccentric</th><th rowspan="2">Inclined-centric</th><th colspan="2">Inclined-eccentric</th></tr>
<tr><th>Positive</th><th>Negative</th></tr>
<tr><td>30° – 34°</td><td>0.40</td><td rowspan="3">0.40</td><td rowspan="2">0.35</td><td>0.65</td></tr>
<tr><td>35° – 36°</td><td>0.45</td><td rowspan="2">0.70</td></tr>
<tr><td>37° – 39°</td><td>0.50</td><td rowspan="2">0.40</td></tr>
<tr><td>40° – 44°</td><td>0.55</td><td>0.45</td><td rowspan="2">0.75</td></tr>
<tr><td>≥ 45°</td><td>0.65</td><td>0.50</td><td>0.45</td></tr>
</table>

Notes:

1) ϕ_f determined from Standard Penetration Test results
2) granular material is assumed to extend below the base of the footing at least two (2.0) times the width of the foundation.
3) The resistance factors were evaluated for a target reliability β_T = 3.0

Conclusion and Summary

- It was found that for the footings of larger sizes (B>3m (9.9ft)), the loading tests were not carried to the failure load
- Biases for the tests in Natural Soil Condition and Controlled Soil Conditions were analyzed separately
- For the footing sizes in similar ranges (0.1m < B ≤ 1.0m), the scatter of bias was more for footings on/in natural soil conditions
- The majority of the relevant data refers to small size foundations (B ≤ 3.3ft (1.0m)) on controlled compacted material. Many of the highway shallow foundations on soils are built on compacted materials and hence, the statistical data of the uncertainty can be used for that purpose
- There appears to be a trend of increase in bias with the footing size within the range of footing sizes available for testing (which seems to conform with the observation made by Vesic (1969))

Conceptual Design – Influence of Serviceability

ϕ's based on Serviceability Limit States

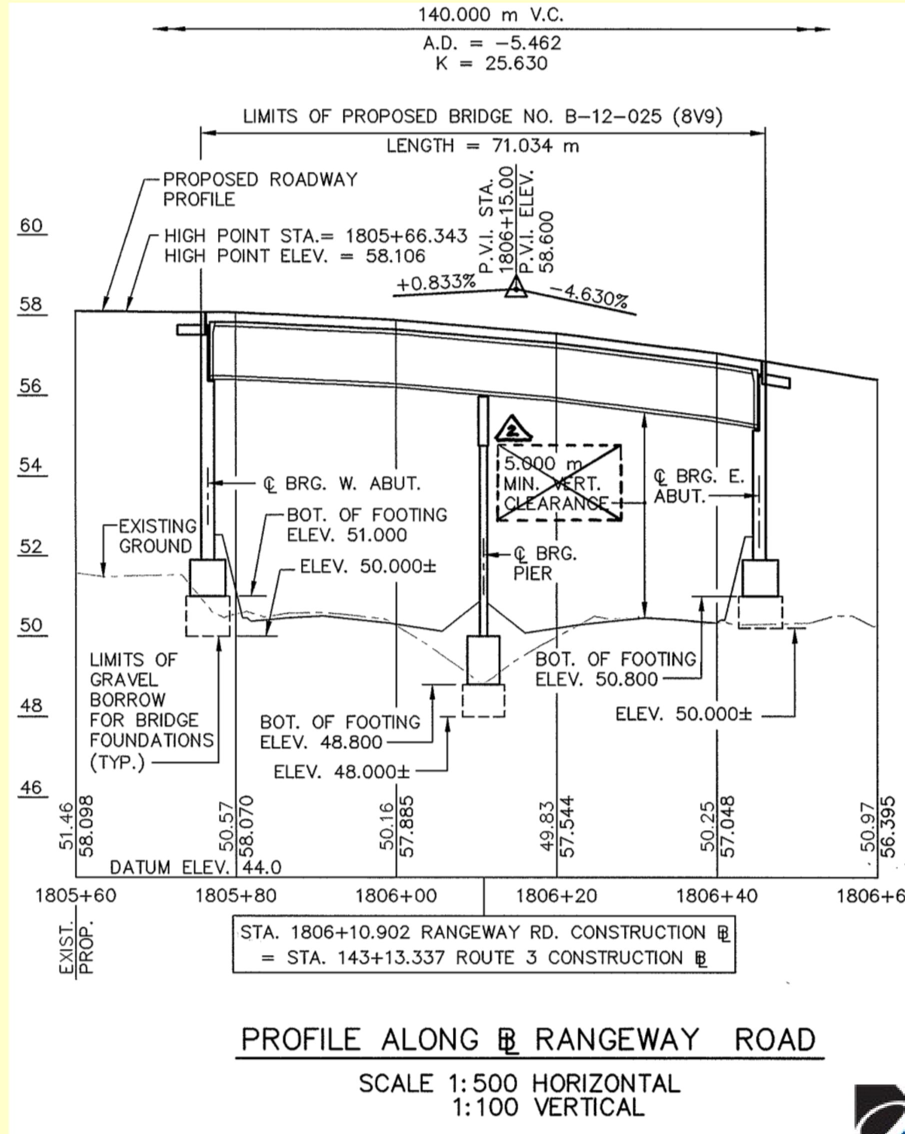
- **Developed as a part of Project NCHRP 12-66**
- **Bias = measured load / calculated load for a given settlement**
- **For reliability index = 1.28 (p_f = 10%), and load factors taken as unity**
- **Bias of LL = 1.15, COV_{QL} = 0.2**
  Bias of DL = 1.05, COV_{QD} = 0.1

Method	Range of Settlement Δ (inch)	Resistance Factor ϕ	Efficiency Factor ϕ/λ
AASHTO	$0.00 < \Delta \leq 1.00$	0.85	0.34
	$1.00 < \Delta \leq 1.50$	0.80	0.48
	$1.50 < \Delta \leq 3.00$	0.60	0.48

Conceptual Design – Granular Soils

Example 2 NCHRP Report 651: Known Load and Settlement

- **Central Pier of a bridge in Billerica (Rangeway Rd over Rte.3) (B-12-025)**
- **Design (factored) load is 3688.3kips for ultimate limit state and 2750kips for service limit state (unfactored)**
- **Allowable settlement 1.5inches**

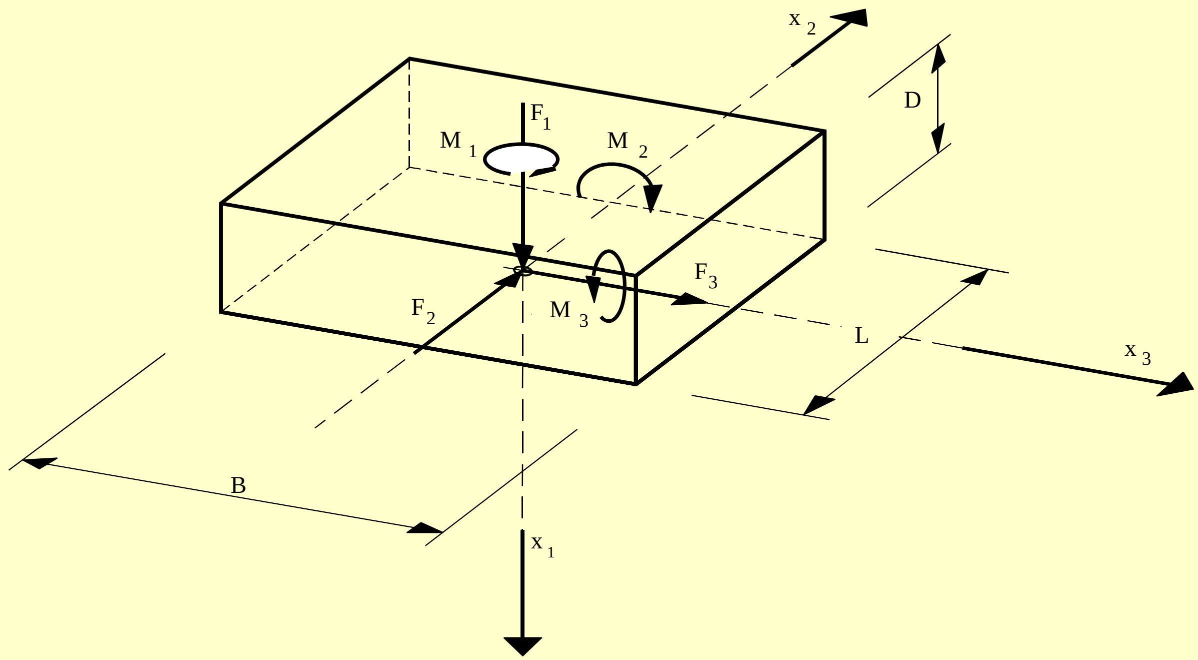


Conceptual Design – Granular Soils

Subsurface condition

- Footing rests on gravel borrow of unit wt 120.0pcf (18.85kN/m^3) and the soil friction angle considered to be 38°, which replaces approx. 3ft of loose granular fill overlaying 5.5ft of coarse sand and gravel underlain by a rock layer
- GWT present at foundation level
- Length of the required foundation = 52.4ft (fixed)

Loading Convention and Notations



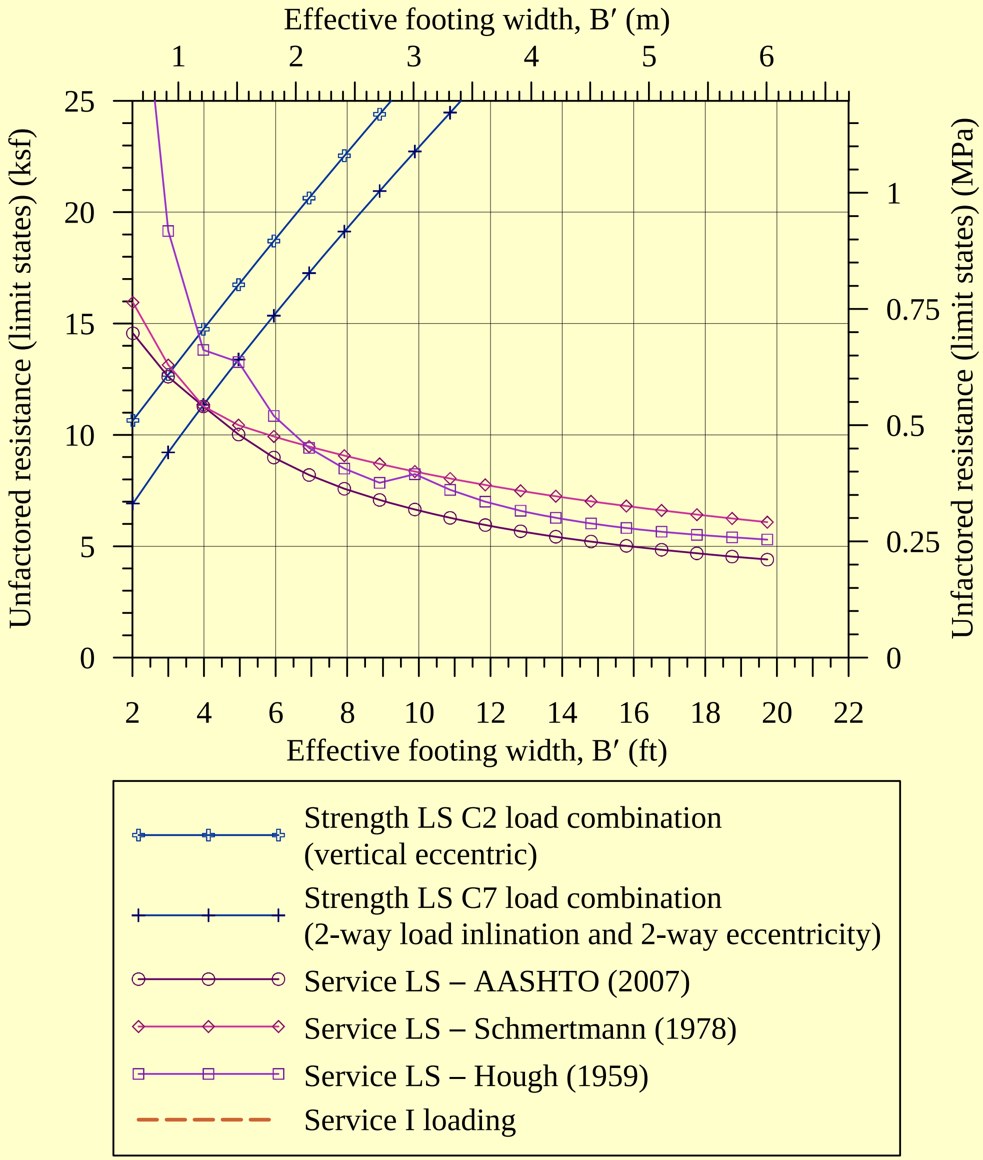
The vertical centric loading is F1; F2 and F3 are horizontal loadings along the transverse (x2-direction or z-direction) and longitudinal (x3-direction or y-direction) directions of the bridge, respectively. M3 is the moment about the longitudinal direction (x3- or y-axis) due to transverse loading and M2 is the moment about the z-axis (transverse direction) due to longitudinal loading. The load eccentricity across the footing width is eB = M2/F1 and across the footing length is eL = M3/F1. The resultant load inclination is given by . $\sqrt{F_2^2 + F_3^2} / F_1$

Conceptual Design – Granular Soils

Unfactored Resistances (ksf)

Figure H-5. Variation of unfactored bearing resistance for Strength-I and Service-I limit states with effective footing width for Example 2 (NCHRP Report 651)

Note: The settlement calculations are done for B and transformed to B'



Conceptual Design – Granular Soils

Unfactored Resistances (kips)

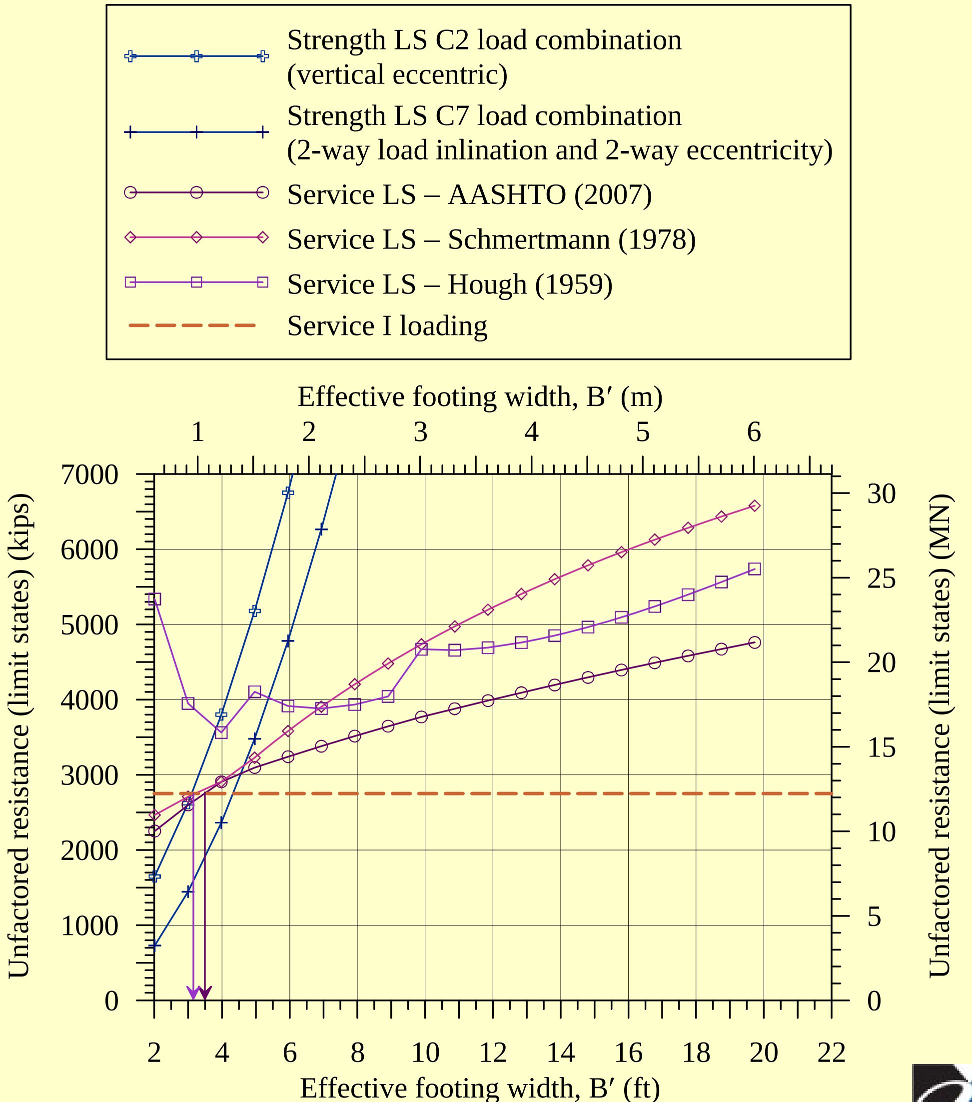


Figure H-5 cont. Variation of unfactored bearing resistance for Strength-I and Service-I limit states with effective footing width for Example 2 (NCHRP Report 651)

Conceptual Design – Granular Soils

Factored Resistances (ksf)

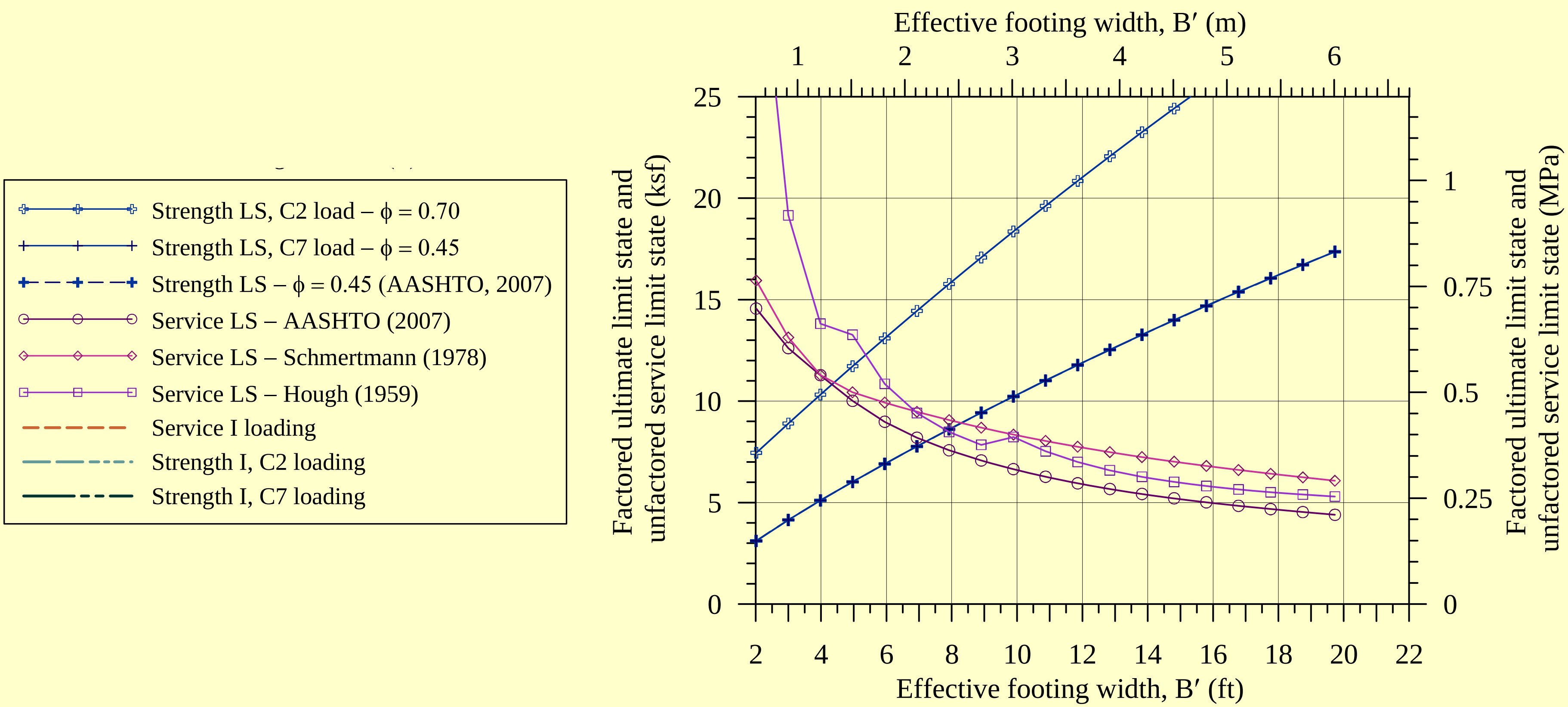
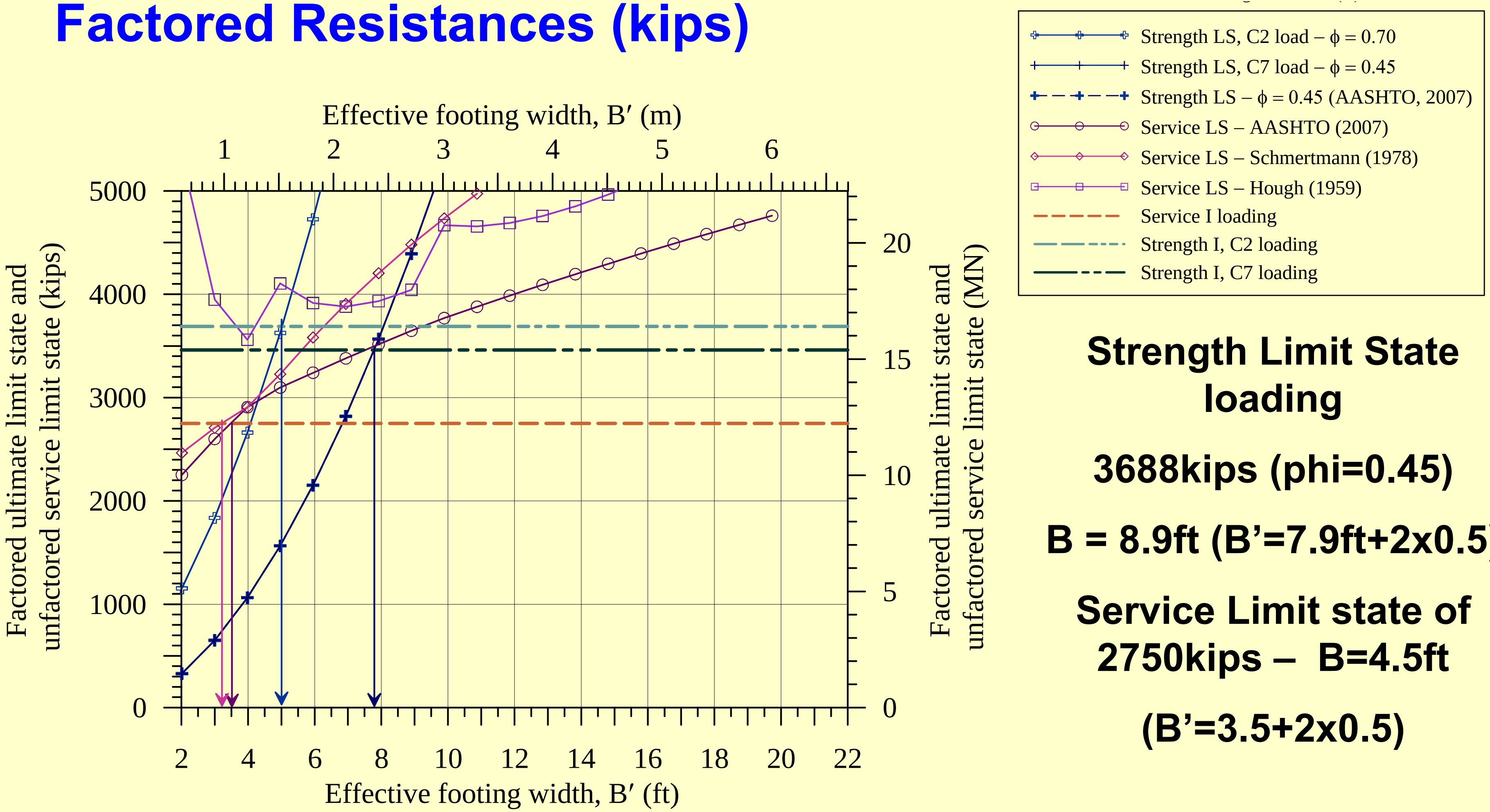


Figure H-6 Variation of factored bearing resistance for Strength-I and unfactored resistance for Service-I limit state with effective footing width for Example 2 (NCHRP Report 651)

Conceptual Design – Granular Soils

Factored Resistances (kips)



Strength Limit State loading

3688kips (phi=0.45)

B = 8.9ft (B'=7.9ft+2x0.5)

Service Limit state of 2750kips – B=4.5ft

(B'=3.5+2x0.5)

Figure H-6 cont. Variation of factored bearing resistance for Strength-I and unfactored resistance for Service-I limit state with effective footing width for Example 2 (NCHRP Report 651)

Intermediate Conclusions

- The Strength Limit State governs the footing dimensions in this design example with a requirement for B=8.9ft vs. B=4.5ft for the service limit state
- The bridge was designed with B=13.1ft most likely due to the differences in design procedures (especially settlement)

BC Shallow Foundations on Rock - OUTLINE

1. **Broad Objectives**
2. **Database UML/GTR RockFound07**
3. **Rock Classification and Properties**
4. **Methods of Analyses Selected for Establishing the Uncertainty in B.C. of Foundations on Rock**
5. **Calibration – evaluation of resistance factors**
6. **Summary and Conclusions**

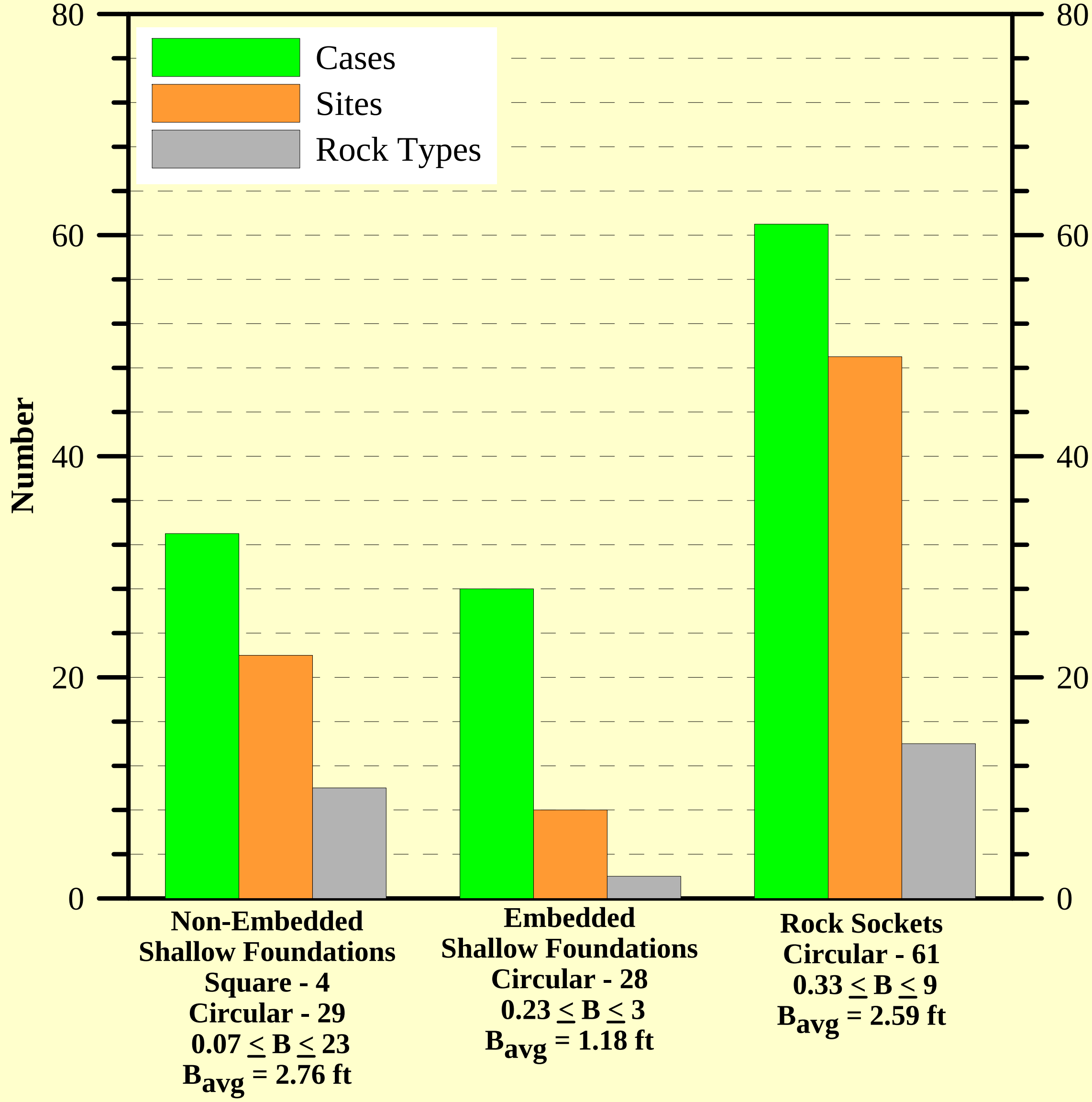
1. Broad Objectives

- Examining the methods for B.C. and displacement evaluation of shallow foundations on rock.
- Establishing the uncertainty of the methods in order to develop the resistance factors.
- In contrast to shallow foundations on soil, the design of shallow foundations on hard rock is by and large controlled by the B.C. and not by settlement. Both however are investigated

2. DATABASE UML/GTR RockFound 07

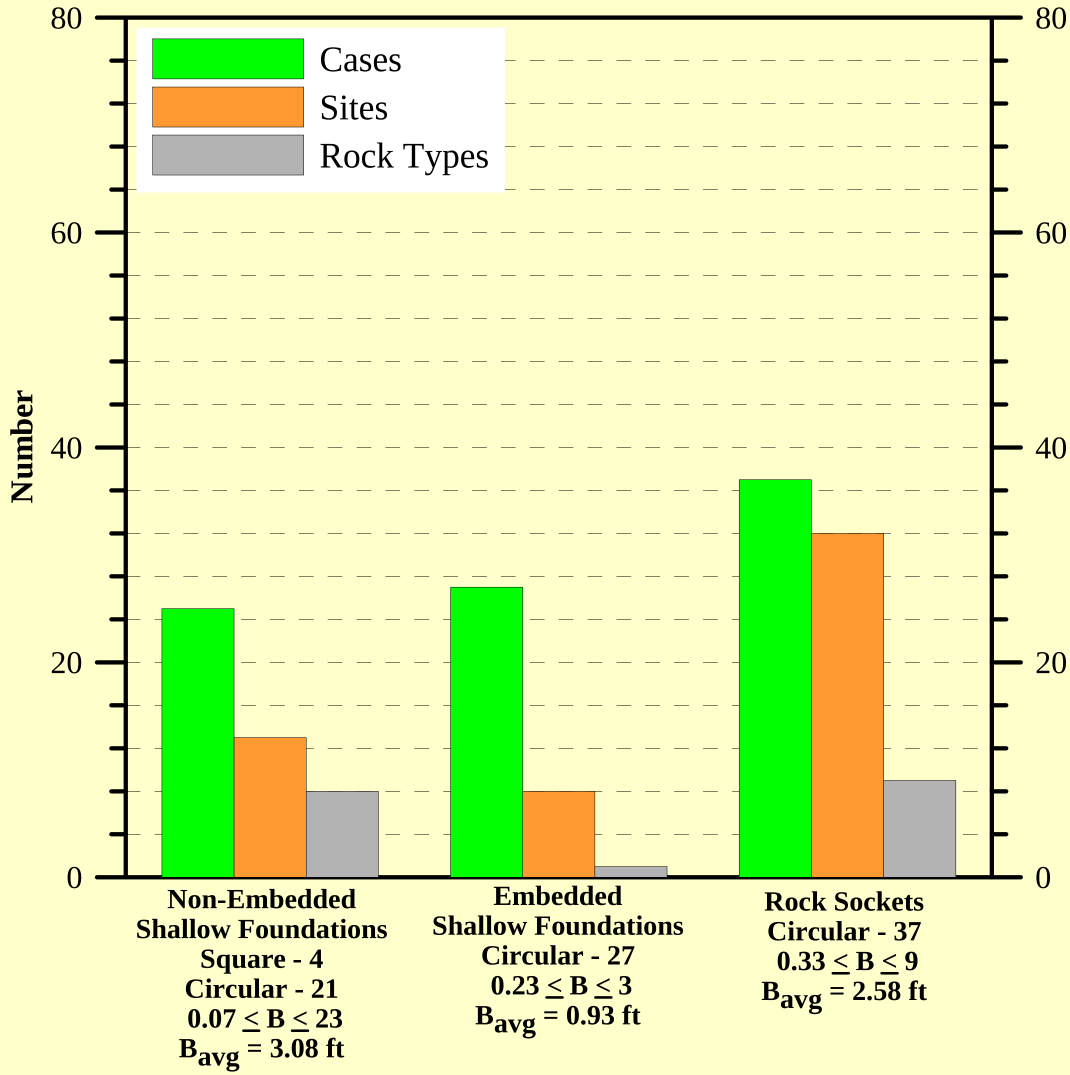
- Comprised of 122 foundation case histories of load tests in/on rock and IGM's.
- The database has 61 footings cases (28 cases D>0, 33 cases D=0) and 61 rock socket cases for which the base behavior (load and displacement) under loading was monitored.
- 89 of the 122 cases were used for the uncertainty determination of the settlement of foundations on rock.

2. Database UML/GTR RockFound07



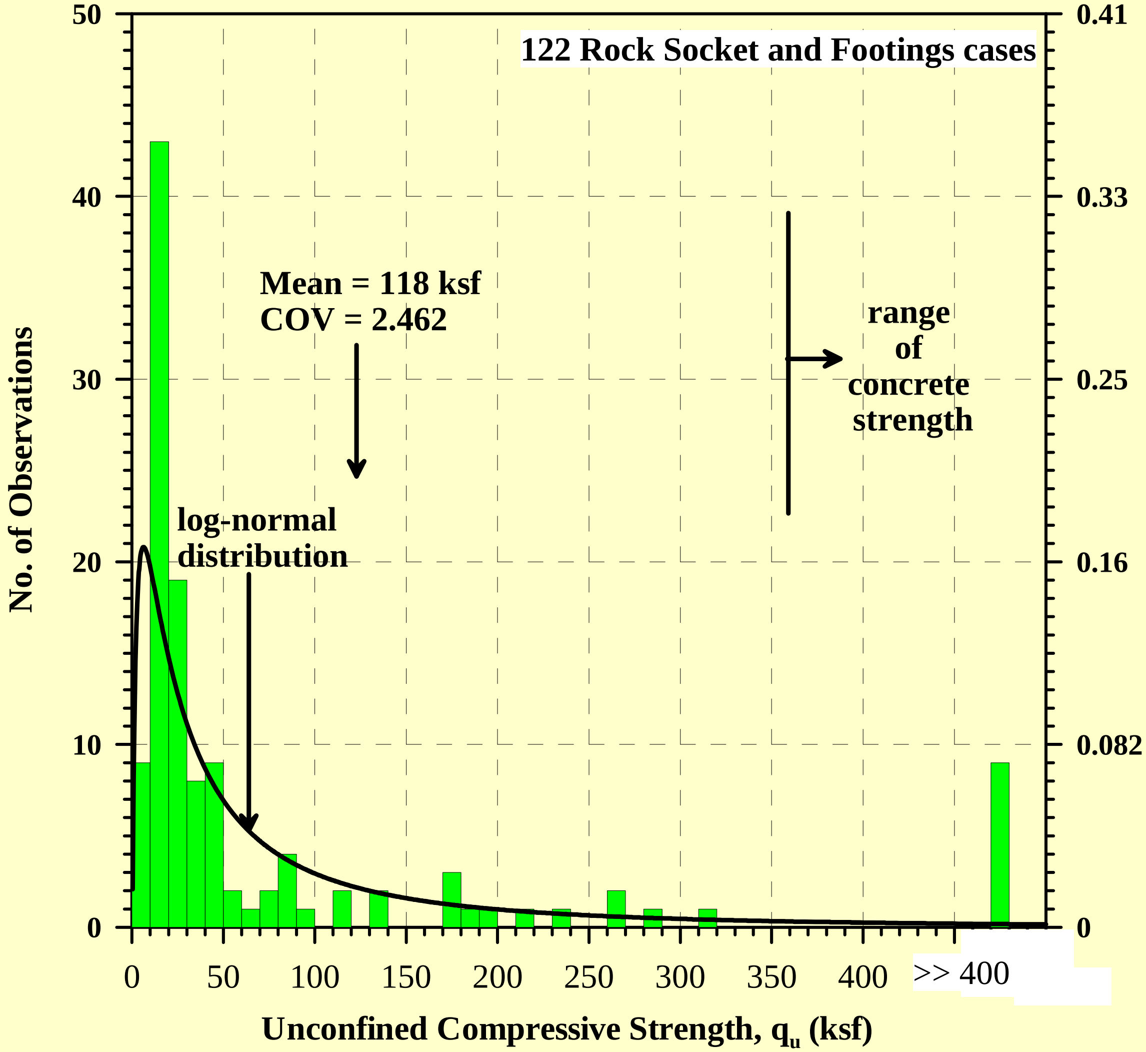
Distribution of Case Histories used in B.C. Analysis

2. Database UML/GTR RockFound07



Distribution of Case Histories used in Settlement Analysis

2. Database UML/GTR RockFound07



AASHTO table 10.4.6.4-1 : Geomechanical Classification of Rock Masses:

Relative rating (for RMR)

Lowest 0

20-70ksf

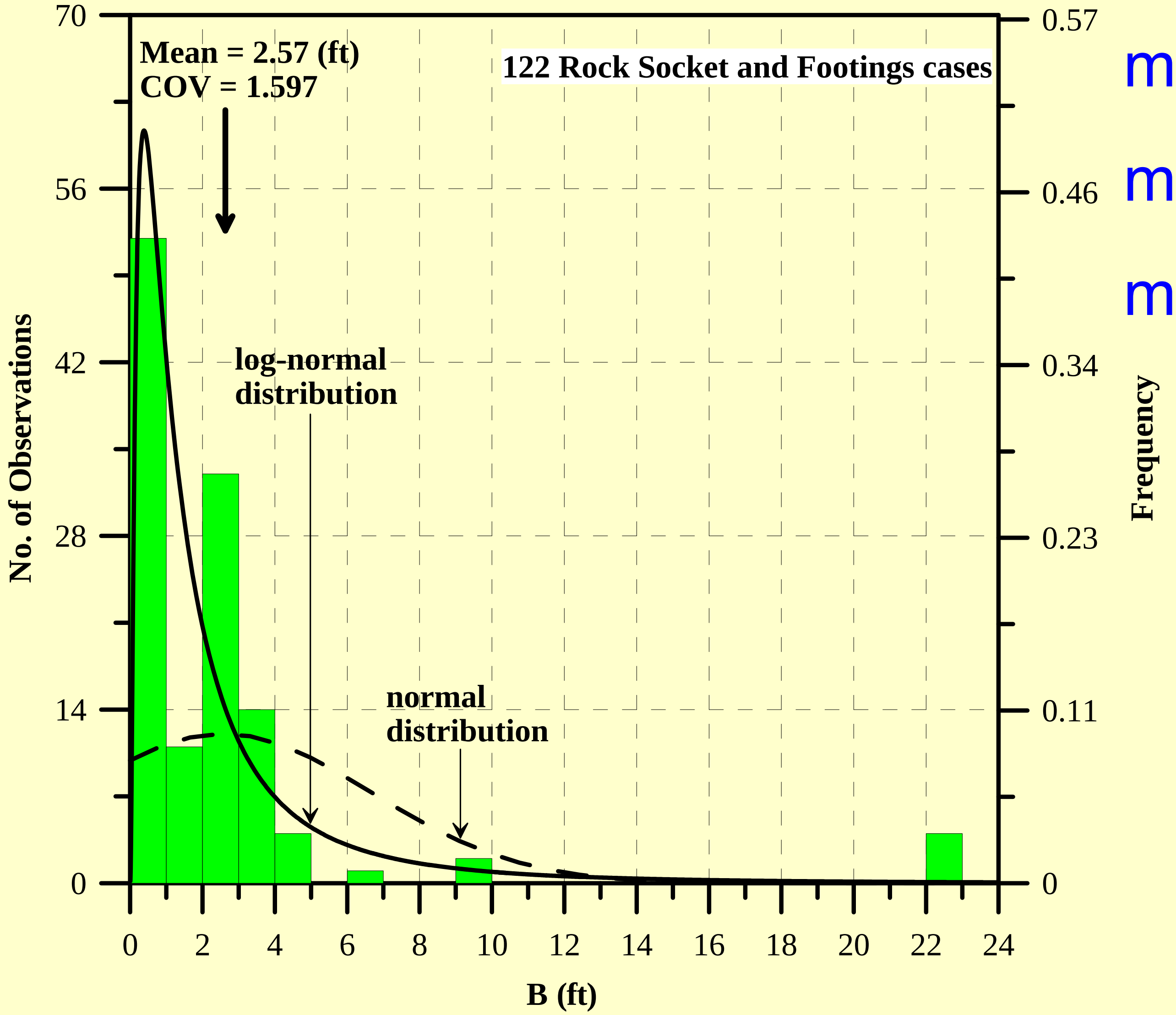
Highest 15

>4320ksf

One definition for IGM q_u < 20ksf

Distribution of the Unconfined Compressive Strength (q_u) for all 122 Case Histories in Database UML/GTR RockFound07

2. Database UML/GTR RockFound07



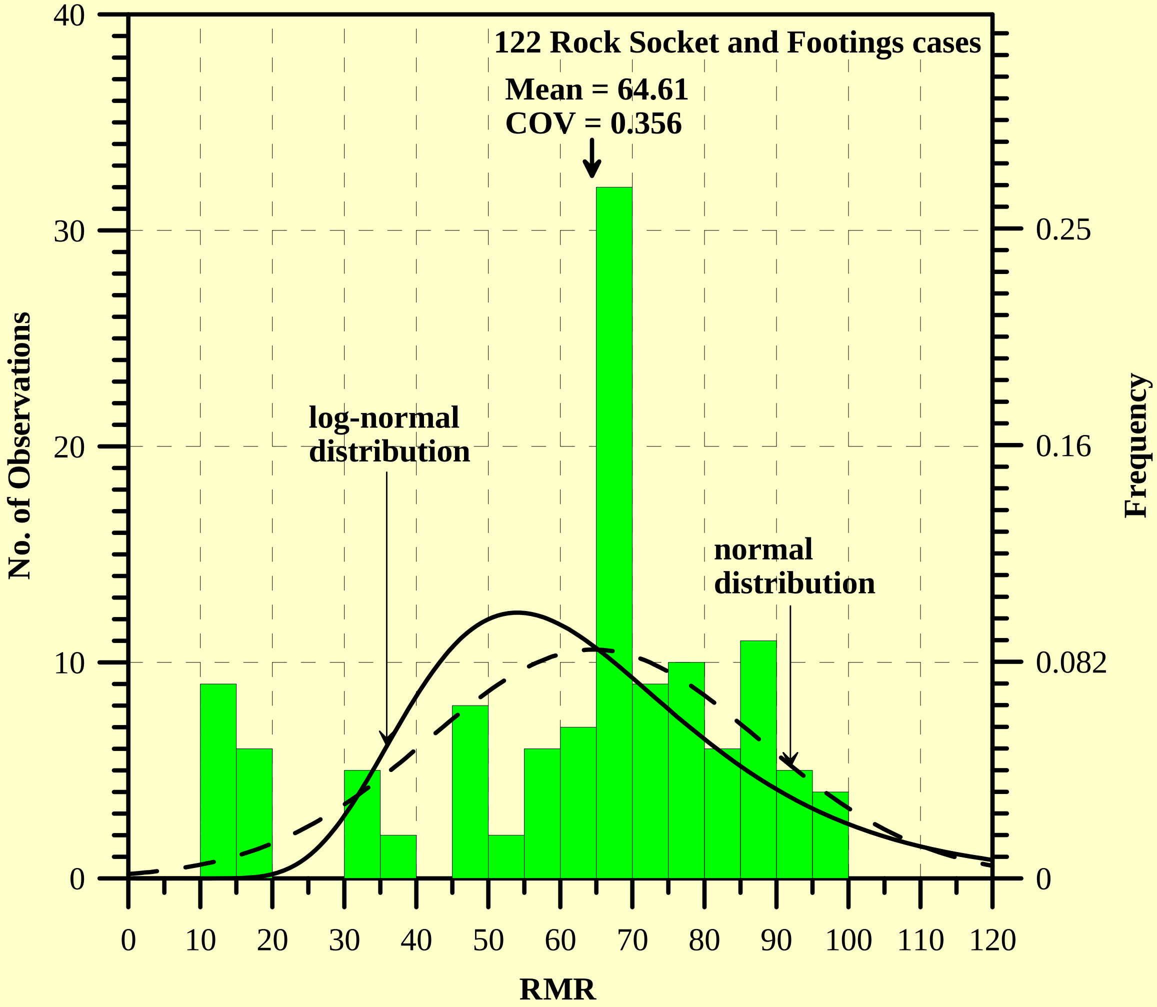
m_B = 3.93ft D=0

m_B = 1.18ft D>0

m_B = 2.47ft RockS

Distribution of the Foundation Width (B) for all 122 Case Histories in Database UML/GTR RockFound07

2. Database UML/GTR RockFound07



m_{RMR} = 65 All

m_{RMR} = 65 D=0

m_{RMR} = 44 D>0

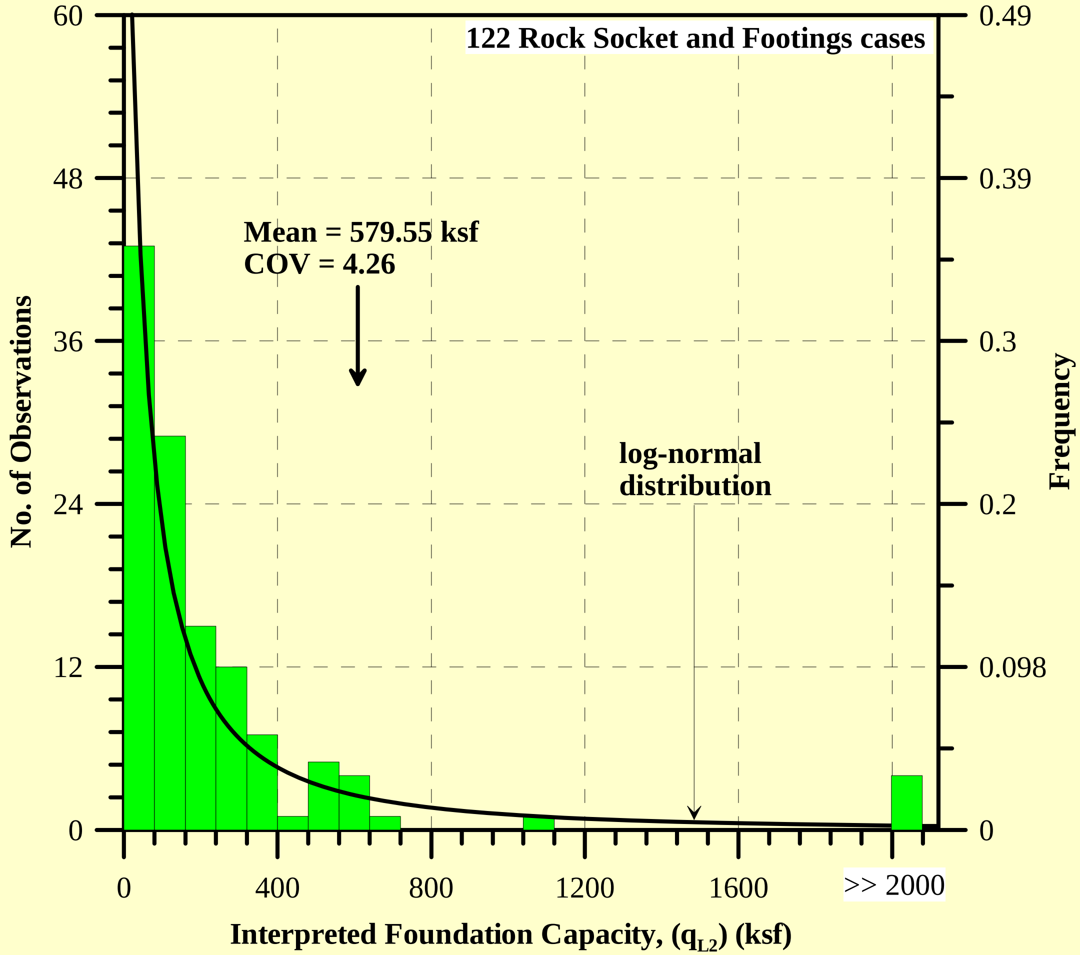
m_{RMR} = 74 RockS

Note

RMR > 85
v. good rock

Distribution of RockMass Rating (RMR) for all 122 Case Histories in Database UML/GTR RockFound07

2. Database UML/GTR RockFound07



m_{qL2} = 580ksf All

m_{qL2} = 1647ksf D=0

m_{qL2} = 51ksf D>0

m_{qL2} = 244ksf RockS

Distribution of qL2 (ksf) for all 122 Case Histories in Database UML/GTR RockFound07

3. Rock Classification and Properties

- **Rock is a natural aggregate of minerals that cannot be readily broken by hand and that will not disintegrate on a first wetting and drying cycle.**
- **A rockmass comprises blocks of intact rock that are separated by discontinuities such as cleavage, bedding planes, joints and faults.**
- **These naturally formed discontinuities create weakness zones within the rockmass, thereby reducing the material strength.**

3. Rock Classification and Properties

Rock is classified with respect to its geological origin or lithology as follows:

- **Igneous rocks, such as granite, diorite and basalt, which are formed by the solidification of molten material, either by intrusion of magma at depth in the earth's crust, or by extrusion of lava at the earth's surface.**
- **Sedimentary rocks, such as sandstone, limestone and shale, which are formed by lithification of sedimentary soils.**
- **Metamorphic rocks, such as quartzite, schist, marble and gneiss, which were originally igneous or sedimentary rocks, and which have been altered physically and sometimes chemically or mineralogically, by the application of intense heat and/or pressure at some time in their geological history.**

3. Rock Classification and Properties

- **The strength and stiffness properties of rockmasses are required in the design of foundations in or on rock. These properties are functions of the properties of the intact rock and the discontinuities.**

- **The two most commonly used rockmass classification systems in Civil engineering are:**
  1. **Rockmass Rating (RMR), Bieniawski, (1974) with several modifications up to 1989 - used in tunneling and foundations, adopted by the International society for Rock mechanics (ISRM) and the South African Council of Scientific and Industrial Research (CSIR).**
  2. **Q-system, Barton et al., 1974 used in tunneling and adopted by the Norwegian Geotechnical Institute index (NGI-index)**

- **In this study, the RMR classification system was adopted because it is most commonly used, it was favored by the available rock property data of the case histories and GSI noted by two states is based on the RMR-system.**

3. Rock Classification and Properties

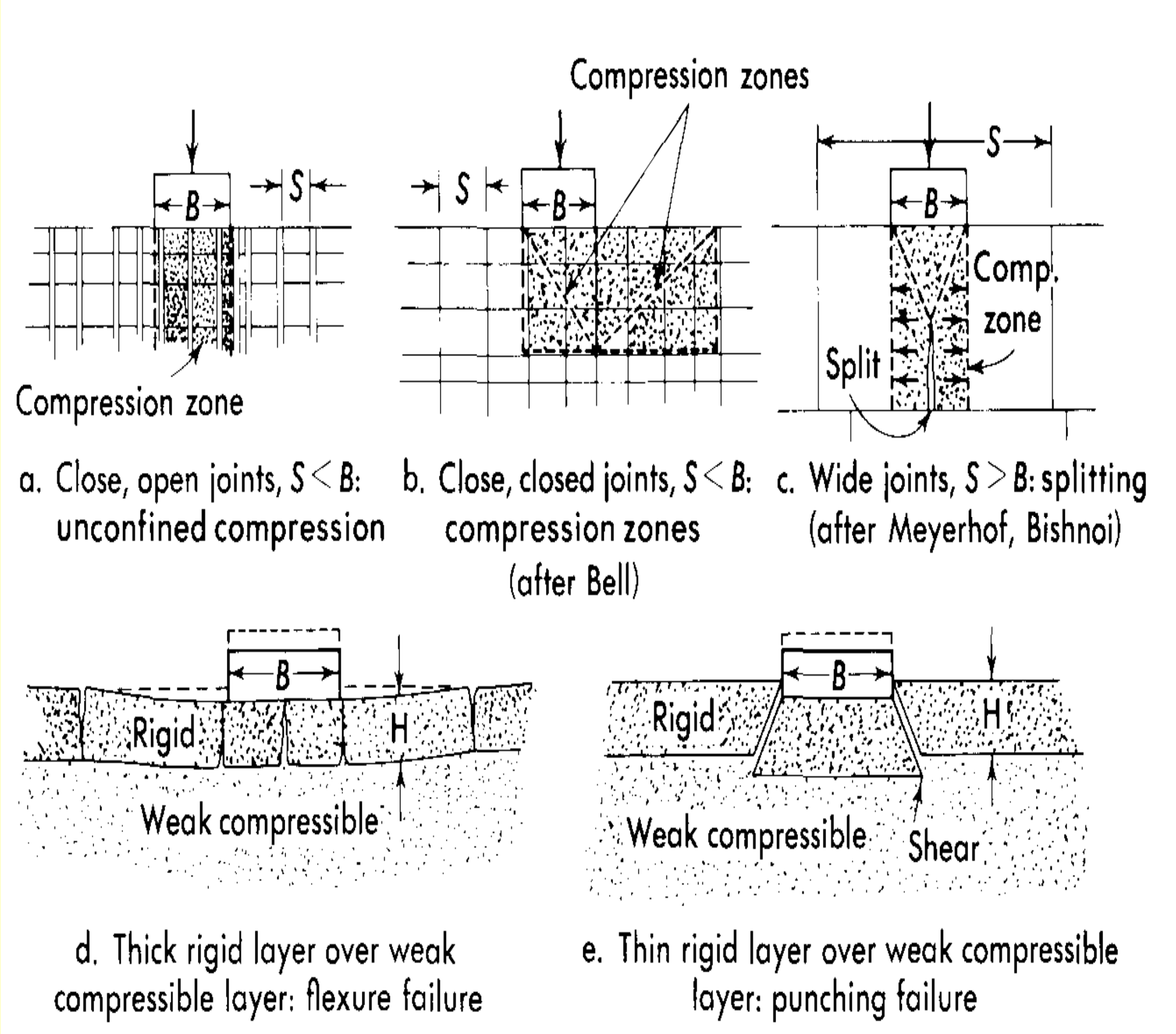
- **The RMR-system uses the following 6 parameters, whose ratings are added to obtain a total RMR-value:**

  i. Unconfined compressive strength of intact rock material (q_u)
  ii. Rock quality designation (RQD)
  iii. Joint or discontinuity spacing (s)
  iv. Joint condition
  v. Ground water condition
  vi. Joint orientation.

  See AASHTO Tables 10.4.6.4-1&2 for the above parameters and relative ratings (5 first and table 2 for vi)

- **Hoek et al. (1995) introduced the GSI-system as a means of estimating the strength and deformation properties of jointed rockmasses. For RMR > 18 the GSI = RMR (Bieniawski, 1976).**

4. Failure Modes of Foundations on Rock



B.C. Failure Modes of Rock (Sowers, 1979)

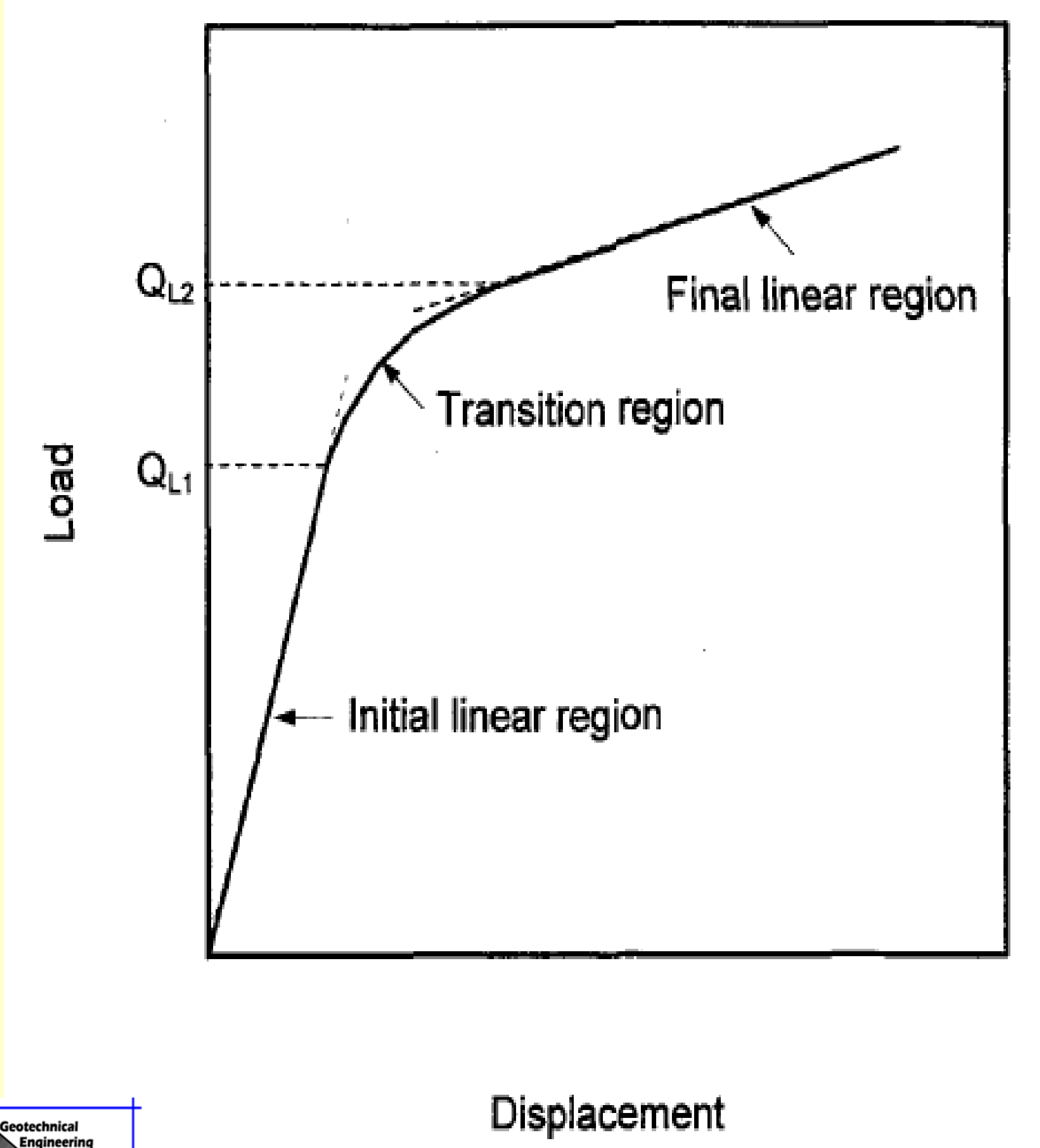
4. Methods of Analyses Selected for Establishing the Uncertainty in B.C. of Foundations on Rock

- **The ratio of the measured and interpreted capacity (q_{L2}) to the calculated B.C. (q_{ult}) (the bias) was used to asses the uncertainty of the different design methods. The calculated B.C. (q_{ult}) was determined in 5 ways, namely:**

  (a) following the semi empirical method by Carter and Kulhawy (1988)
  (b) following the analytical method by Goodman (1989)
  (c) following the Hoek and Brown (1980) failure criterion
  (d) utilizing the N_c^* and q_u based on the relationship developed by Zhang and Einstein's (1998) and examined in this study
  (e) following relationships between measured or interpreted B.C. (q_{L2}) and q_u developed in this study as a function of rockmass quality utilizing AASHTO (2007) RMR ranges

- **Only (a) and (b) are presented in NCHRP research report 651**
- **The margin of safety of the AASHTO (2007) presumptive values was also examined.**

Hirany and Kulhawy (1988) Failure criterion



Hirany and Kulhawy (1988) proposed the L1-L2 method for interpreting the "failure" load or "ultimate" capacity of foundations from load-displacement curves.

The unique peak or asymptote value in the curves is taken as the measured or interpreted capacity (QL2=qL2).

For 79 cases qL2 could be evaluated, 43 cases are based on reported failure load.

4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock

Using the Hoek-Brown strength criterion, Carter and Kulhawy (1988) developed the curved strength envelope represented by Equation 1 for B.C. evaluation of jointed rockmasses:

$$\sigma_1 = \sigma_3 + \left(mq_u\sigma_3 + sq_u^{\ 2}\right)^{0.5} \qquad (1)$$

in which

- s_1 = major principal effective stress
- s_3 = minor principal effective stress
- q_u = unconfined compression strength of the intact rock
- s and m = empirically determined strength parameters for the rockmass, which are to some degree analogous to c and ϕ of the Mohr-Coulomb failure criterion

4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock

Using the limit-equilibrium approach, Carter and Kulhawy (1988) developed a lower bound to the B.C. for strip and circular footings on jointed rock masses presented below.

$$q_{ult} = \left(m + \sqrt{s}\right) q_u \qquad (2)$$

4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock

Summary of the statistics for the Ratio of Measured to Calculated B.C. using Carter and Kulhawy's (1988) Method

Cases	n	No. of Sites	Mean of Bias m_λ	Standard deviation σ_λ	COV_λ
All Foundations	119	78	8.00	9.92	1.240
All rock sockets	61	49	4.29	3.08	0.716
All footings	58	29	11.90	12.79	1.075

Sub-categorization showed that the more detailed rock measurements are available, the lower the uncertainty.

e.g. 39 Rock Socket cases with measured discontinuity spacing had a COV_λ = 0.93.

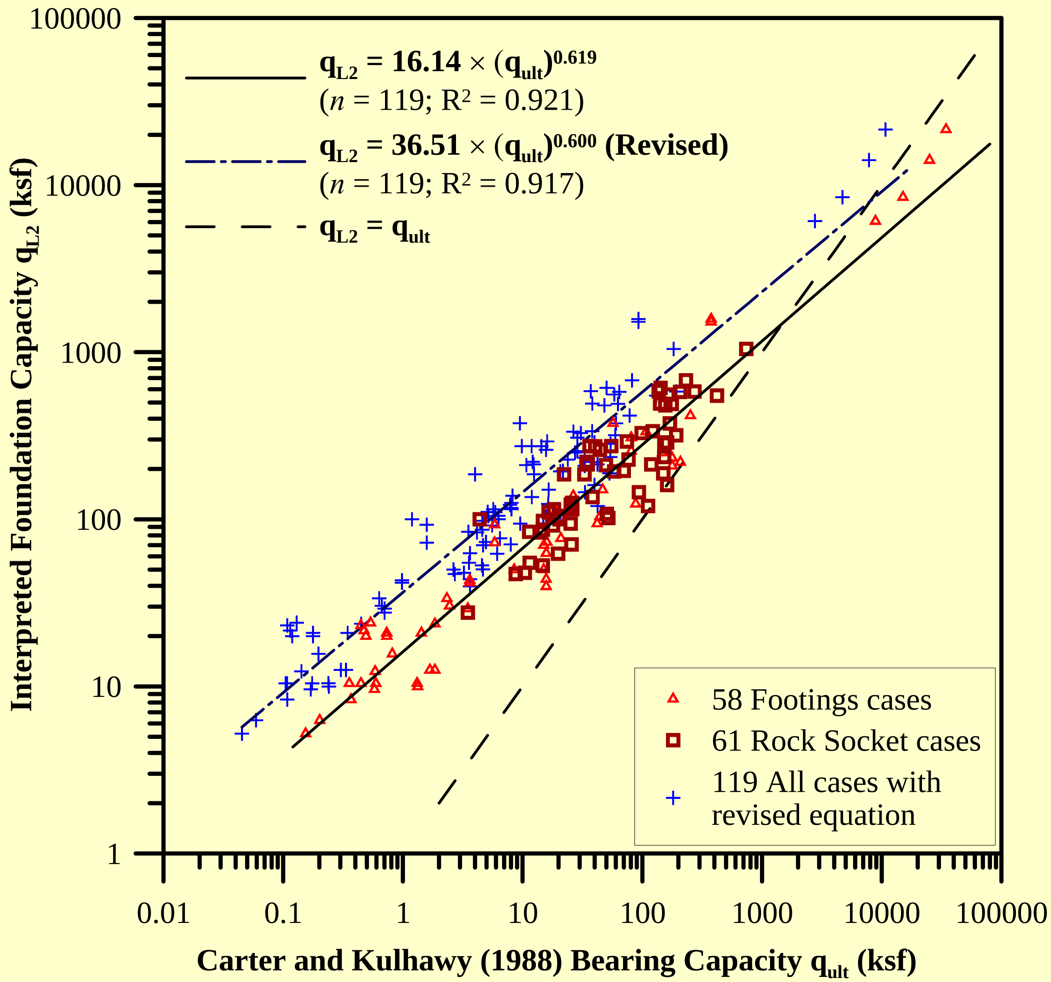
4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock

Table 38 Summary of the statistics for the ratio of measured (q_{L2}) to calculated bearing capacity (q_{ult}) of rock sockets and footings on rock using Carter and Kulhawy (1988) method

Cases	n	No. of Sites	m_λ	σ_λ	COV
All rock sockets	61	49	4.29	3.08	0.716
All rock sockets on fractured rock	11	6	5.26	1.54	0.294
All rock sockets on non-fractured rock	50	43	4.08	3.29	0.807
Rock sockets on non-fractured rock with measured discontinuity spacing (s')	34	14	3.95	3.75	0.949
Rock sockets on non-fractured rock with s' based on AASHTO (2007)	16	13	4.36	2.09	0.480
All footings	58	29	11.90	12.794	1.075
All footings on fractured rock	9	3	2.58	2.54	0.985
All footings on non-fractured rock	49	26	13.62	13.19	0.969
Footings on non-fractured rock with measured discontinuity spacing (s')	29	11	15.55	14.08	0.905
Footings on non-fractured rock with s' based on AASHTO (2007)	20	11	10.81	11.56	1.069

n = number of case histories m_λ = mean of biases σ_λ = standard deviation
COV = coefficient of variation Calculated capacity based on equation (82a)

4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock



Relationship between Carter and Kulhawy (1988) calculated bearing capacity (q_{ult}) using two variations (equations 82a and 82b) and the interpreted bearing capacity (q_{L2}).

4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock

Table 39 Summary of the statistics for the ratio of measured (q_{L2}) to calculated bearing capacity (q_{ult}) using Carter and Kulhawy (1988) method categorized by the rock quality and foundation type

Foundation type	Cases	*n*	No. of Sites	m_λ	σ_λ	COV
All	RMR ≥ 85	23	23	2.93	1.908	0.651
	65 ≤ RMR < 85	57	36	3.78	1.749	0.463
	44 ≤ RMR < 65	17	10	8.83	5.744	0.651
	3 ≤ RMR < 44	22	9	23.62	13.550	0.574
Rock Sockets	RMR ≥ 85	16	16	3.42	1.893	0.554
	65 ≤ RMR < 85	35	24	3.93	1.769	0.451
	44 ≤ RMR < 65	9	8	6.82	6.285	0.921
	3 ≤ RMR < 44	1	1	8.39	--	--
Footings	RMR ≥ 85	7	7	1.81	1.509	0.835
	65 ≤ RMR < 85	22	13	3.54	1.732	0.489
	44 ≤ RMR < 65	8	5	11.09	4.391	0.396
	3 ≤ RMR < 44	21	8	24.34	13.440	0.552

n = number of case histories; m_λ = mean of biases; σ_λ = standard deviation;
COV = coefficient of variation; Calculated capacity based on equation (82a)

4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock

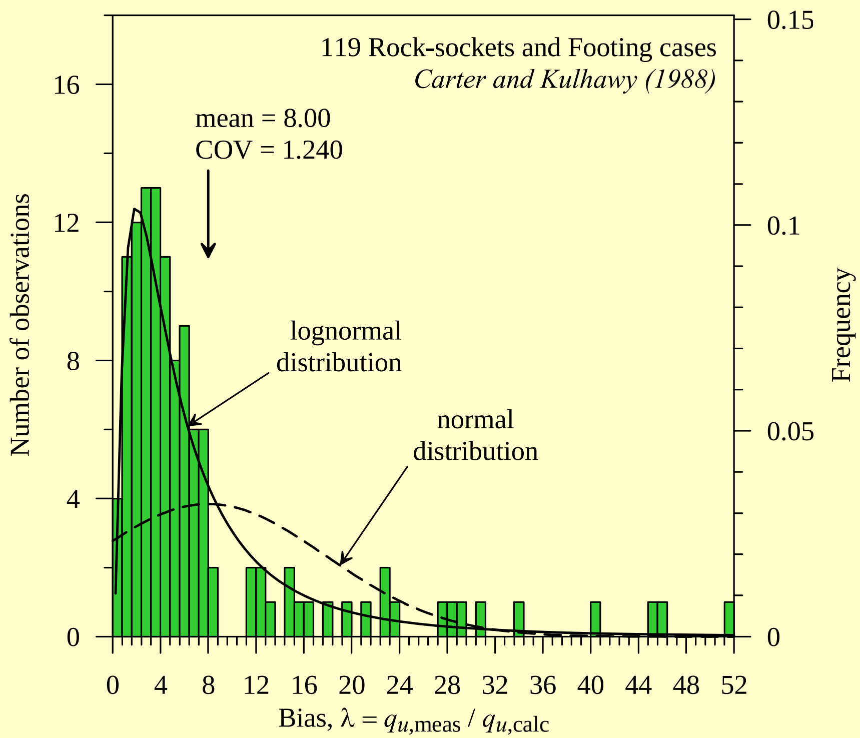


Figure 76. Distribution of the ratio of the interpreted bearing capacity (q_{L2}) to the bearing capacity (q_{ult}) calculated using Carter and Kulhawy's (1988) method (equation 82a) for the rock sockets and footings in database UML-GTR RockFound07.

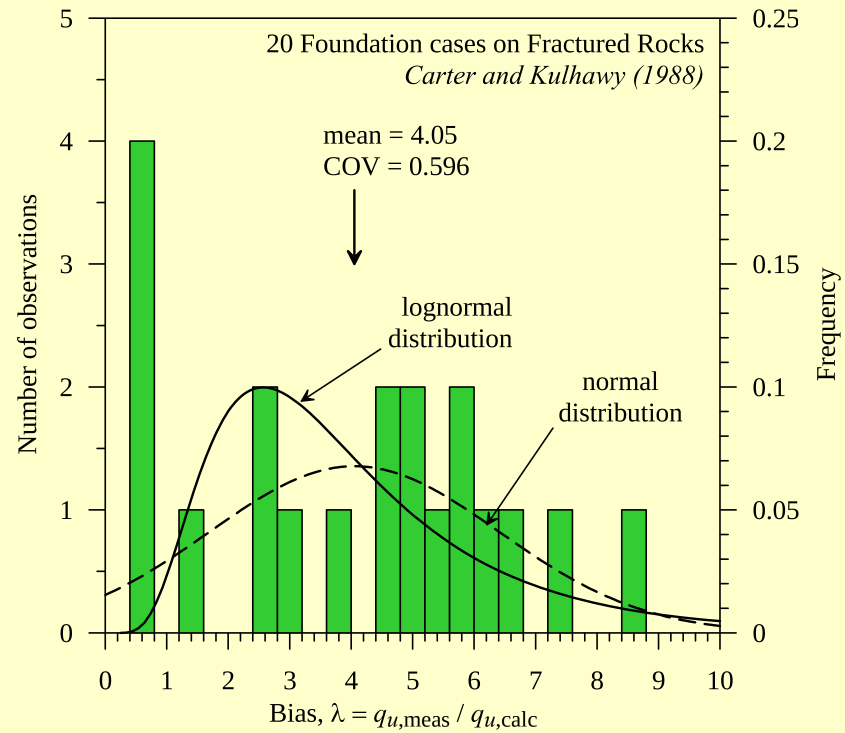


Figure 77. Distribution of the ratio of the interpreted bearing capacity (q_{L2}) to the bearing capacity (q_{ult}) calculated using Carter and Kulhawy's (1988) method (equation 82a) for foundations on fractured rock in database UML-GTR RockFound07.

4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock

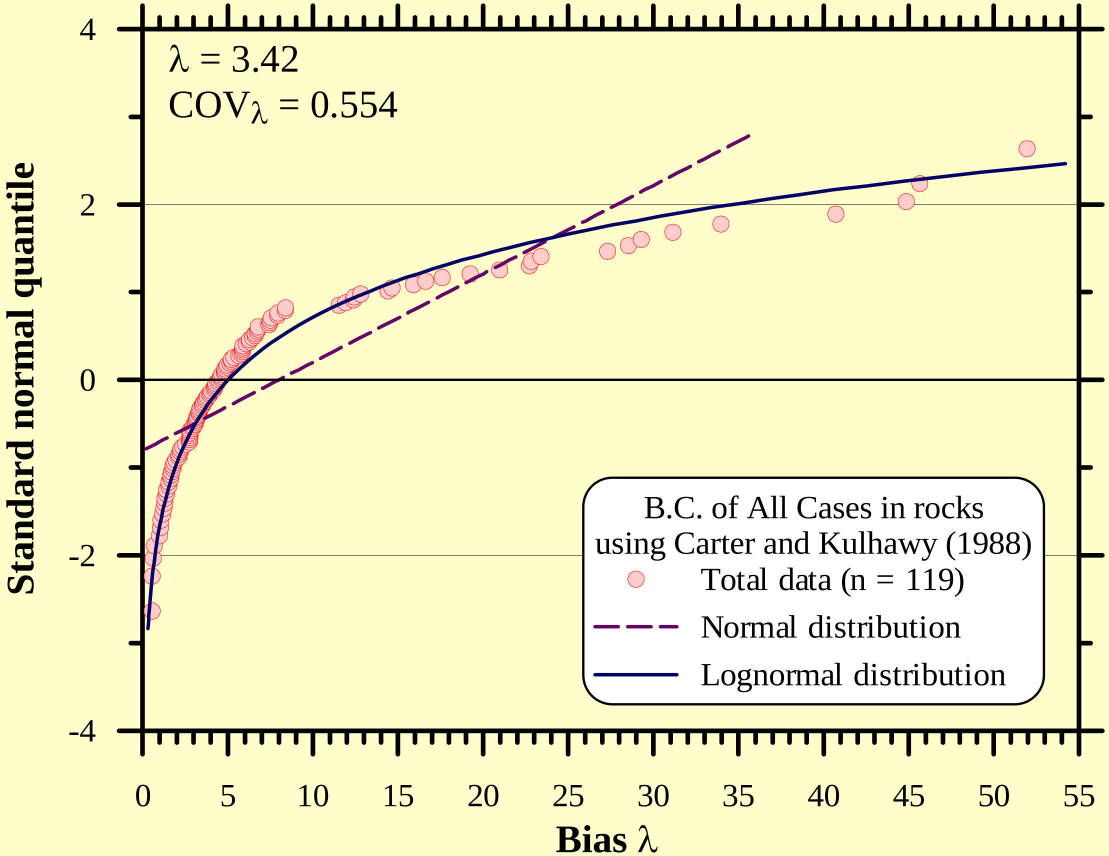


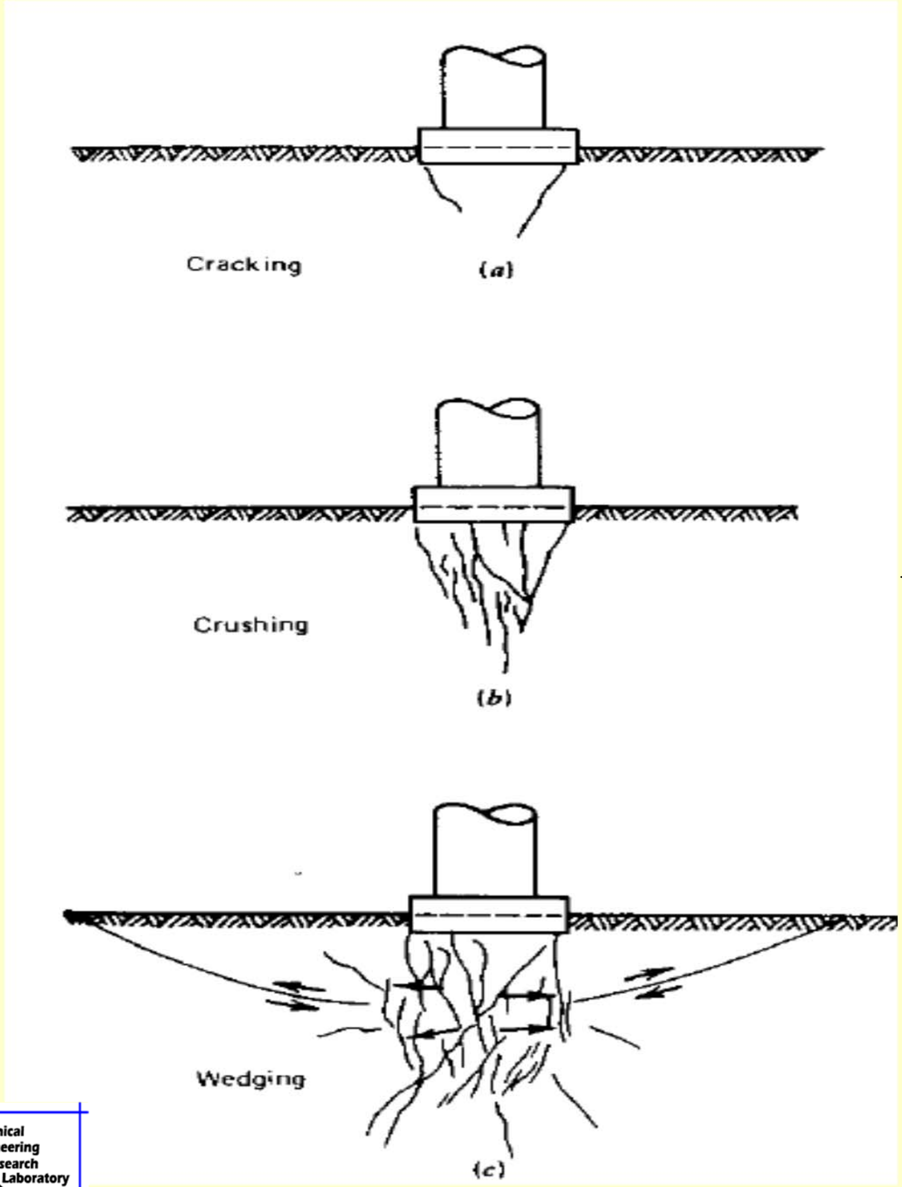
Figure 117. Comparison of the unfiltered bias for BC calculated using Carter and Kulhawy (1988) method for total cases in/on rocks in the database and the theoretical normal and lognormal distributions.

4a Carter and Kulhawy (1988) - B.C. of Foundations on Rock

Table 69 Calibrated resistance factors for different datasets of resistance bias obtained using Carter and Kulhawy's (1988) method

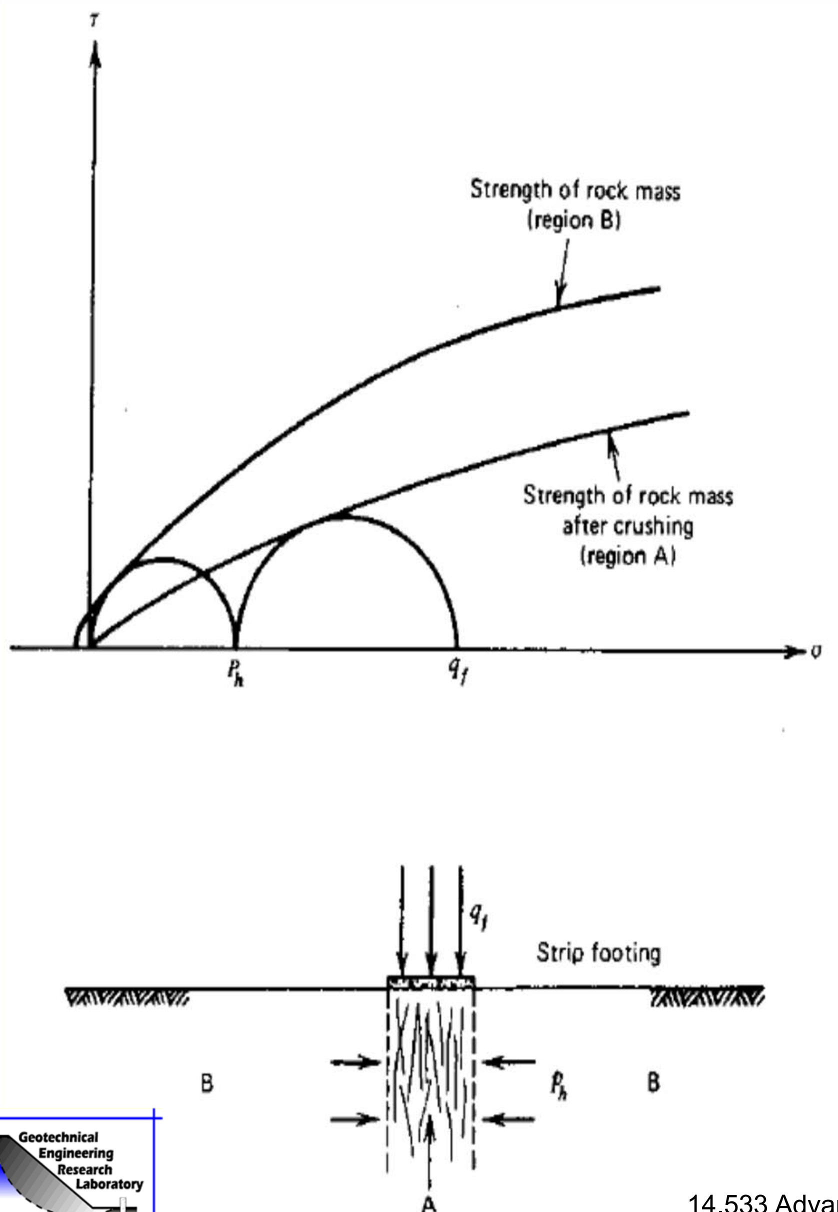
Dataset	No. of cases	Bias		Resistance factor ϕ ($\beta_T = 3$)	
		Mean λ	COV_λ	MCS	Recommended
All cases	119	8.00	1.240	0.372	0.35
RMR ≥ 85	23	2.93	0.651	0.535	0.50
65 ≤ RMR < 85	57	3.78	0.463	1.149	1.00
44 ≤ RMR < 65	17	8.83	0.651	1.612	1.00
3 ≤ RMR < 44	22	23.62	0.574	5.295	1.00

4b Goodman (1989) - B.C. of Foundations on Rock



Goodman (1989) considered the mode of failure presented in a through c, in which a laterally expanding zone of crushed rock under a strip footing induces radial cracking of the rock on either side.

4b Goodman (1989) - B.C. of Foundations on Rock



Strength of crushed rock under footing – lower envelope.

Strength of the less fractured neighboring rock – upper envelope.

P_h in the figure is equal to q_u of the adjacent rock (Zone B) which is the largest confining stress that can be mobilized to support the rock under the footing (Zone A).

The figure suggests that B.C. of a homogeneous discontinuous rockmass can not be less than the q_u of the rockmass around the footings and this can be taken as the lower bound.

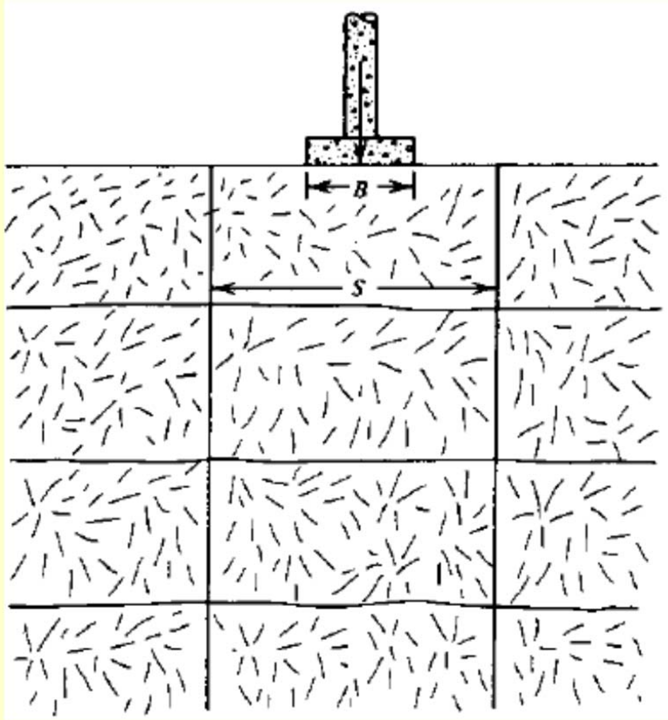
4b Goodman (1989) - B.C. of Foundations on Rock

The lower bound is represented by the following Equation:

$$q_{ult} = q_u\left(N_\phi + 1\right) \quad (3)$$

in which

$$N_\phi = \tan^2\left(45 + \phi/2\right) \quad (4)$$



Goodman (1989) developed the B.C. Equation 5 for footings resting on orthogonal vertical joints each spaced distance s in which lateral stress transfer is nil.

$$q_{ult} = q_u\left\{\frac{1}{N_\phi - 1}\left[N_\phi\left(\frac{S}{B}\right)^{(N_\phi - 1)/N_\phi} - 1\right]\right\} \quad (5)$$

4b Goodman (1989) - B.C. of Foundations on Rock

Summary of the statistics for the Ratio of Measured to Calculated B.C. using Goodman's (1989) Method

Cases	n	No. of Sites	Mean of Bias m_λ	Standard Deviation σ_λ	COV_λ
All Foundations	119	78	1.35	0.72	0.535
All rock sockets	61	49	1.52	0.82	0.541
All footings	58	29	1.23	0.66	0.539

Sub-categorization suggests that if more details of rock measurements are available, the uncertainty is reduced.

1. 34 Rock Socket cases with measured discontinuity spacing had a COV_l = 0.48.
2. 8 Rock Socket cases with measured discontinuity spacing and friction angle had a COV_l = 0.18.

4b Goodman (1989) - B.C. of Foundations on Rock

Table 40 Summary of the statistics for the ratio of measured (q_{L2}) to calculated bearing capacity (q_{ult}) of rock sockets and footings on rock using Goodman (1989) method

Cases	n	No. of Sites	m_λ	σ_λ	COV
All	119	78	1.35	0.72	0.535
Measured discontinuity spacing (s') and friction angle (ϕ_f)	67	43	1.51	0.69	0.459
Measured discontinuity spacing (s')	83	48	1.43	0.66	0.461
Measured friction angle (ϕ_f)	98	71	1.41	0.76	0.541
Fractured	20	9	1.24	0.34	0.276
Fractured with measured friction angle (ϕ_f)	12	7	1.33	0.25	0.189
Non-fractured	99	60	1.37	0.77	0.565
Non-fractured with measured s' and measured ϕ_f	55	37	1.55	0.75	0.485
Non-fractured with measured discontinuity spacing (s')	63	39	1.49	0.72	0.485
Non-fractured with measured friction angle (ϕ_f)	86	64	1.42	0.81	0.569
Spacing s' and ϕ_f, both based on AASHTO (2007)	5	3	0.89	0.33	0.368
Discontinuity spacing (s') based on AASHTO (2007)	36	21	1.16	0.83	0.712
Friction angle (ϕ_f) based on AASHTO (2007)	21	7	1.06	0.37	0.346

n = number of case histories m_λ = mean of biases σ_λ = standard deviation COV = coefficient of variation

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4b Goodman (1989) - B.C. of Foundations on Rock

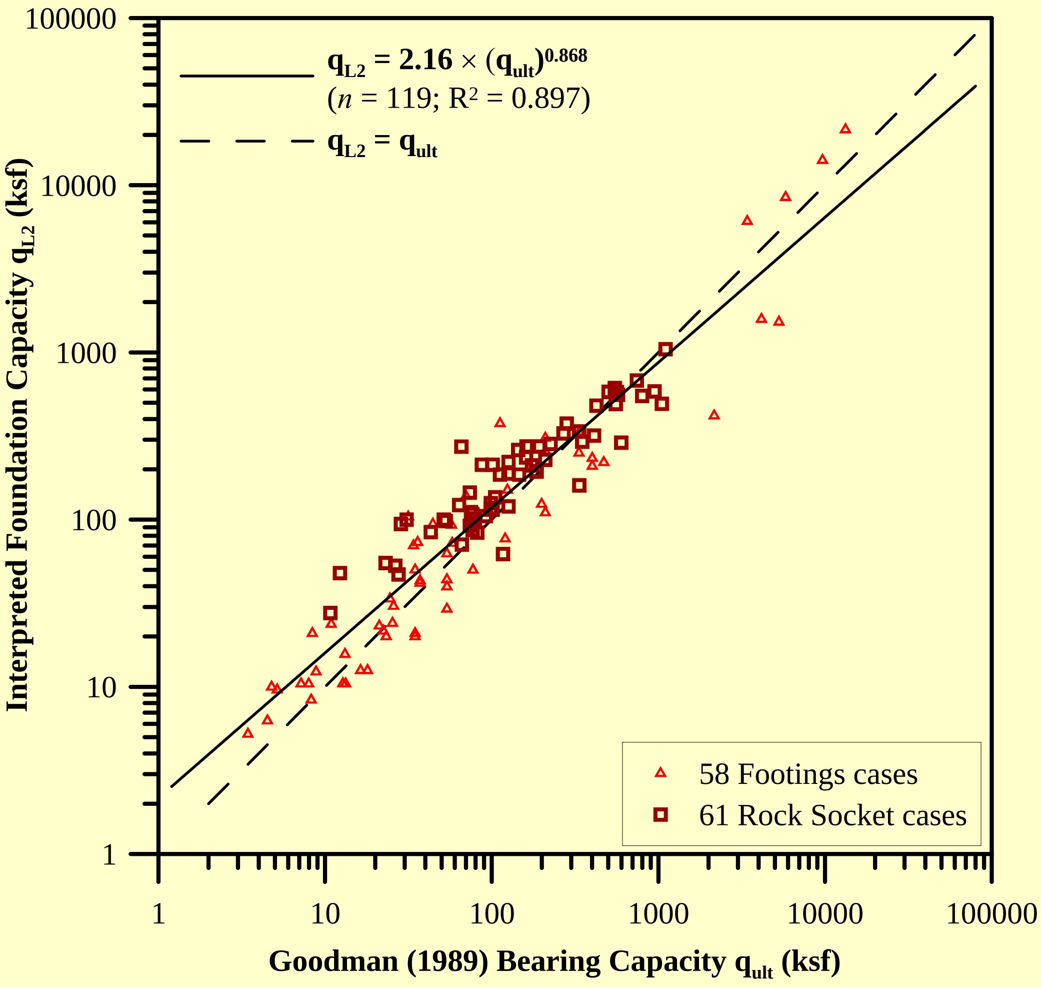

Figure 78. Relationship between Goodman's (1989) calculated bearing capacity (q_{ult}) and the interpreted bearing capacity (q_{L2}).

4b Goodman (1989) - B.C. of Foundations on Rock

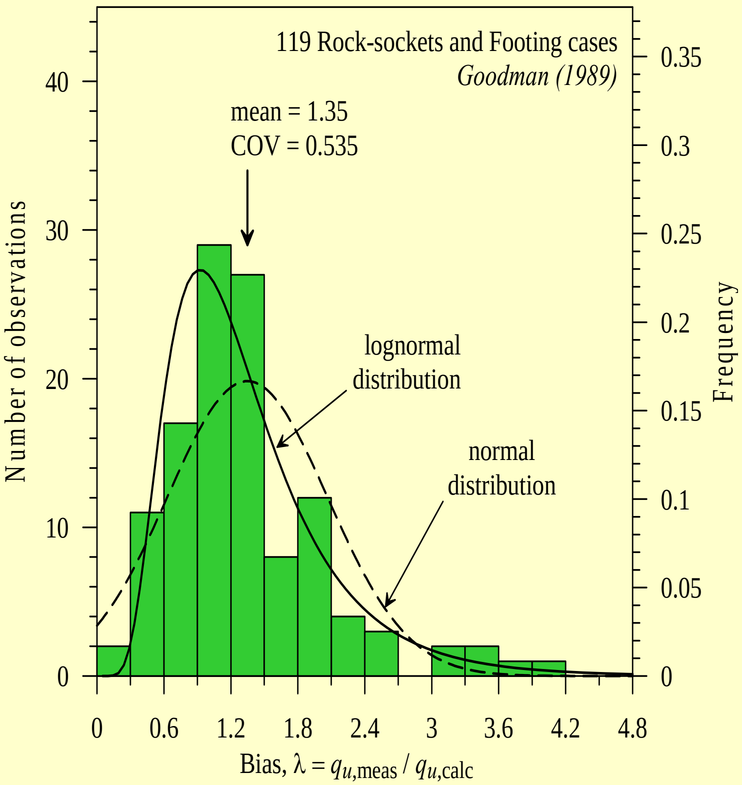

Figure 79. Distribution of the ratio of the interpreted bearing capacity (q_{L2}) to the bearing capacity (q_{ult}) calculated using Goodman's (1989) method for the rock sockets and footings in database UML-GTR RockFound07.

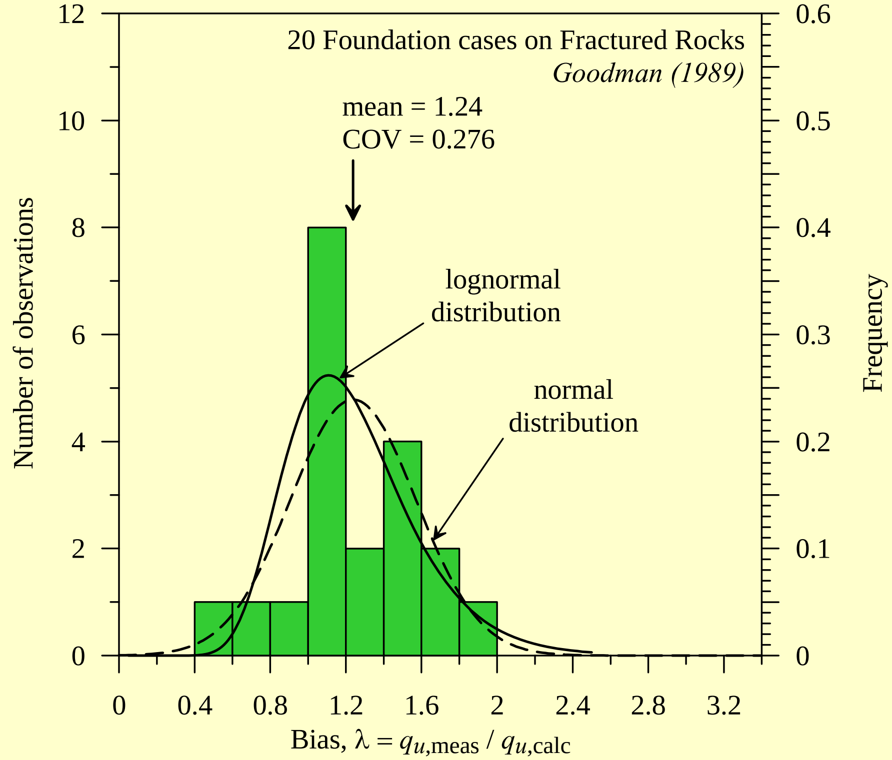

Figure 80. Distribution of the ratio of the interpreted bearing capacity (q_{L2}) to the bearing capacity (q_{ult}) calculated using Goodman's (1989) method for foundations on fractured rock in database UML-GTR RockFound07

4b Goodman (1989) - B.C. of Foundations on Rock

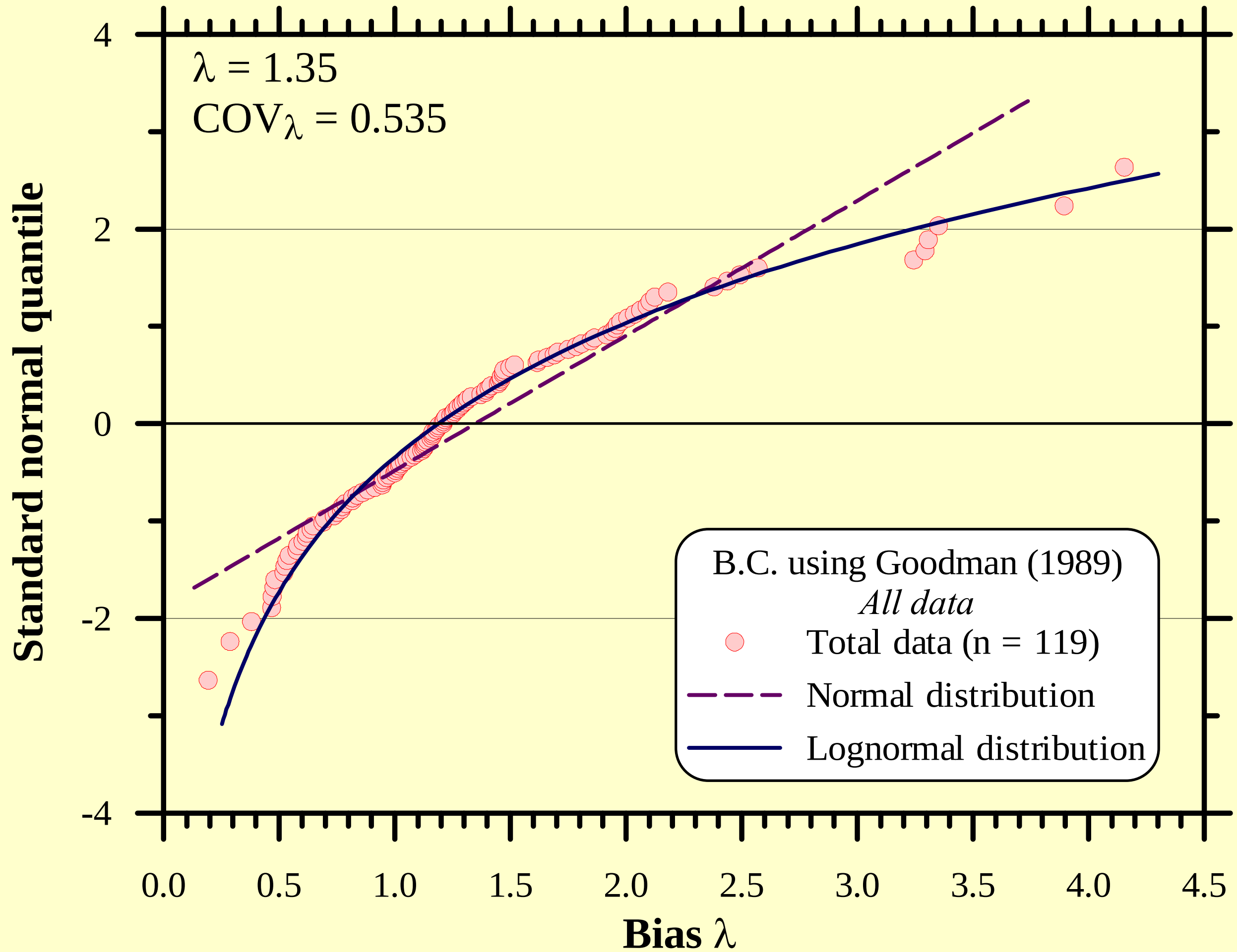

Figure 113. Comparison of the unfiltered bias for BC calculated using Goodman (1989) method for all data and the theoretical normal and lognormal distributions.

4b Goodman (1989) - B.C. of Foundations on Rock

Table 42 Summary of the statistics for the ratio of measured (q_{L2}) to calculated bearing capacity (q_{ult}) using Goodman (1989) method categorized by the rock quality

Foundation type	Cases	*n*	No. of Sites	m_λ	σ_λ	COV
All	RMR ≥ 85	23	23	1.55	0.679	0.438
	65 ≤ RMR < 85	57	36	1.33	0.791	0.595
	44 ≤ RMR < 65	17	10	1.27	0.746	0.586
	3 ≤ RMR < 44	22	9	1.24	0.529	0.426
Rock Sockets	RMR ≥ 85	16	16	1.59	0.809	0.509
	65 ≤ RMR < 85	35	24	1.40	0.722	0.515
	44 ≤ RMR < 65	9	8	1.47	0.916	0.624
	3 ≤ RMR < 44	1	1	1.27	--	--
Footings	RMR ≥ 85	7	7	1.46	0.204	0.140
	65 ≤ RMR < 85	22	13	1.22	0.896	0.738
	44 ≤ RMR < 65	8	5	1.06	0.461	0.437
	3 ≤ RMR < 44	21	8	1.24	0.542	0.437

n = number of case histories; m_λ = mean of biases; σ_λ = standard deviation; COV = coefficient of variation

4b Goodman (1989) - B.C. of Foundations on Rock

Table 68 Calibrated resistance factors for different datasets of resistance bias obtained using Goodman's (1989) method

Dataset	No. of cases	Bias		Resistance factor ϕ ($\beta_T = 3$)	
		Mean λ	COV_λ	MCS	Recommended
All data	119	1.35	0.535	0.336	0.30
Measured friction angle ϕ_f	98	1.41	0.541	0.346	0.35
Measured spacing s′	83	1.43	0.461	0.437	0.40
Measured friction angle ϕ_f and s′	67	1.51	0.459	0.464	0.45

5. Calibration of Resistance Factors

Outline of Major Points

Both Monte Carlo (MC) simulation and the First Order Second Moment (FOSM) methods were used for the resistance factors calculations. The resistance factors are mostly in the range of 0.1 to 0.5. The resistance factors of Carter and Kulhawy (1988) are greater than one under three categories due to extremely high bias. Although theoretically there is no restriction for the resistance factor magnitude, practically it often leads to misconception as to the economics of a method, as will be further explained.

The tables include the number of case histories and sites, mean bias and COV for each examined method of analysis and its application procedure or subcategory. For each of the three examined target reliabilities, the following is presented:

1. **Rounded resistance factor based on the values initially evaluated.**
2. **Efficiency factor, a measure evaluating the relative efficiency of each design method with the higher value representing a more effective method. Such measure is required as often design engineers evaluate the economic value of a design method by the absolute value of the factor of safety or resistance factor (e.g. lower F.S. or higher ϕ, representing a more 'efficient' method). A discussion and presentation of the concept are presented by Paikowsky et al. (2004).**

5. Calibration of Resistance Factors

Table 70 Recommended resistance factors for foundations in/on rock based on β_T = 3.0 (p_f = 0.135%)

Method of Analysis	Equation	Application	ϕ	Efficiency Factor ϕ/λ (%)
Carter and Kulhawy (1988)	$q_{ult} = q_u\left(m + \sqrt{s}\right)$	All	0.35	4.4
		RMR ≥ 85	0.50	17.1
		65 ≤ RMR < 85	1.00	26.5
		44 ≤ RMR < 65		11.3
		3 ≤ RMR < 44		4.2
Goodman (1989)	For fractured rocks: $q_{ult} = q_u\left(N_\phi + 1\right)$ For non-fractured rocks: $q_{ult} = q_u\left(\frac{1}{N_\phi - 1}\left\{N_\phi\left(\frac{s'}{B}\right)^{(N_\phi - 1)/N_\phi} - 1\right\}\right)$	All	0.30	22.2
		Measured ϕ_f	0.35	24.8
		Measured s′	0.40	28.0
		Measured s′ and ϕ_f	0.45	29.8


THANK YOU VERY MUCH FOR YOUR ATTENTION