

PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work. You may work together, but everyone must turn in his/her own homework set.

1. Pinchover and Rubinstein problem 1.1 Show that each of the following equations has a solution of the form $u(x, y) = f(ax + by)$ for a proper choice of constants a, b . Find the constants for each example.

(a) $u_x + 3u_y = 0$

(b) $3u_x - 7u_y = 0$

(c) $2u_x + \pi u_y = 0$

2. Pinchover and Rubinstein problem 1.2 Show that each of the following equations has a solution of the form $u(x, y) = e^{\alpha x + \beta y}$. Find the constants α, β for each example.

(a) $u_x + 3u_y + u = 0$

(b) $u_{xx} + u_{yy} = 5e^{x-2y}$

(c) $u_{xxxx} + u_{yyyy} + 2u_{xxyy} = 0$

3. Pinchover and Rubinstein problem 2.1 Solve the equation $u_x + u_y = 1$ with the initial condition $u(x, 0) = f(x)$.

4. Pinchover and Rubinstein problem 2.2 Solve the equation $xu_x + (x + y)u_y = 1$ with the initial condition $u(1, y) = y$. Is the solution defined everywhere?

5. Pinchover and Rubinstein problem 2.7

Solve the Cauchy problem equation $u_x + u_y = u^2$, $u(x, 0) = 1$.

FOR STUDENTS ENROLLED IN 92.545.

6. Pinchover and Rubinstein problem 1.3

(a) Show that there exists a unique solution for the system

$$u_x = 3x^2y + y$$

$$u_y = x^3 + x$$

together with the initial condition $u(0, 0) = 0$.

(b) Prove that the system

$$u_x = 2.999999x^2y + y$$

$$u_y = x^3 + x$$

has no solution at all.

7. Pinchover and Rubinstein problem 2.4 Consider the equation $yu_x - xu_y = 0$ ($y > 0$). Check for each of the following initial conditions whether the problem is solvable. If it is solvable, find a solution. If it is not, explain why.

(a) $u(x, 0) = x^2$

(b) $u(x, 0) = x$

(c) $u(x, 0) = x, x > 0$