

PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work. You may work together, but everyone must turn in his/her own homework set.

1. Consider the pde $u_t + (4u^3 + 4u)u_x = 0$.

a) Express the pde in conservation form: $u_t + \frac{\partial}{\partial x} [F(u)] = 0$.

b) Consider a weak solution in the form of a shock wave: $u = \begin{cases} u^- & \text{if } x < \gamma(t) \\ u^+ & \text{if } x > \gamma(t) \end{cases}$

Find the speed of propagation $\gamma'(t)$ of the shock wave.

2. Use D'Alembert's Formula to solve the following Cauchy problem.

$$\begin{aligned} u_{tt} - 9u_{xx} &= 0 && \text{on } -\infty < x < \infty, t > 0 \\ u(x, 0) &= 0 \\ u_t(x, 0) &= 2xe^{-x^2} \end{aligned}$$