92.445/545 Partial Differential Equations Spring 2013
Homework Assignment # 3
Due March 4

PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work. You may work together, but everyone must turn in his/her own homework set.

1. Solve the following Cauchy problem on a semi-infinite domain.

$$\begin{aligned} u_{tt} - 4u_{xx} &= 0 \qquad \text{on } 0 < x < \infty, t > 0 \\ u(x,0) &= xe^{-x} \\ u_t(x,0) &= 0 \\ u(0,t) &= 0 \end{aligned}$$

2. Solve the following Cauchy problem on a semi-infinite domain.

$$\begin{aligned} u_{tt} - 9u_{xx} &= 0 \qquad \text{on } 0 < x < \infty, t > 0 \\ u(x,0) &= 0 \\ u_t(x,0) &= \cos(x) \\ u_x(0,t) &= 0 \end{aligned}$$

FOR STUDENTS ENROLLED IN 92.545.

3. Suppose u is a solution of the wave equation $u_{tt} - c^2 u_{xx} = 0$ on $-\infty < x < \infty, t > 0$ and suppose that $u \to 0, u_x \to 0$, and $u_t \to 0$ as $x \to \pm\infty$. Show that $E = \int_{-\infty}^{\infty} \left(u_t^2 + c^2 u_x^2\right)\, dx$ is constant.

Hints: Use the wave equation and look at $\dfrac{\partial}{\partial x}\left(u_t u_x\right)$