92.445/545 Partial Differential Equations Spring 2013
Homework Assignment # 4
Due April 1

PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work. You may work together, but everyone must turn in his/her own homework set.

1. (Pinchover and Rubinstein problem 3.1) Find the canonical form of the following pde. Be sure to show the change of coordinates that reduces the pde to canonical form.

$$u_{xx} - 6u_{xy} + 9u_{yy} = xy^2$$

2. (Pinchover and Rubinstein problem 5.1) Solve the following IBVP.

$$\begin{aligned} u_t &= 17u_{xx} \qquad \text{on } 0 < x < \pi, t > 0 \\ u(0,t) &= 0 \quad t \geq 0 \\ u(\pi,t) &= 0 \quad t \geq 0 \\ u(x,0) &= \begin{cases} 0 & 0 \leq x \leq \pi/2 \\ 2 & \pi/2 < x \leq \pi \end{cases} \end{aligned}$$

3. (Pinchover and Rubinstein problem 5.4) Solve the following IBVP. Hint: Use a trig identity to replace $\sin^3(x)$ by an equivalent expression.

$$\begin{aligned} u_{tt} &= u_{xx} \qquad \text{on } 0 < x < \pi, t > 0 \\ u(0,t) &= 0 \quad t \geq 0 \\ u(\pi,t) &= 0 \quad t \geq 0 \\ u(x,0) &= \sin^3(x) \quad 0 \leq x \leq \pi \\ u_t(x,0) &= \sin(2x) \quad 0 \leq x \leq \pi \end{aligned}$$

FOR STUDENTS ENROLLED IN 92.545.

4. (Pinchover and Rubinstein problem 3.12) Consider the pde $u_{xx} + yu_{yy} = 0$.

a. Find the domain on which the given pde is elliptic.

b. Find the canonical form of the given pde on the domain you found in part a.