92.445/545 Partial Differential Equations Spring 2013
Homework Assignment # 6
Due April 17

PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work. You may work together, but everyone must turn in his/her own homework set.

1. Solve the following BVP for Laplace's equation.

$$\begin{aligned} \Delta u &= 0 && \text{on } 0 < x < \pi, 0 < y < \pi \\ u(x,0) &= \sin(2x) && 0 < x < \pi \\ u(x,\pi) &= 0 && 0 < x < \pi \\ u(0,y) &= \sin(y) && 0 < y < \pi \\ u(\pi,y) &= 0 && 0 < y < \pi \end{aligned}$$

2. (Pinchover & Rubinstein, problem 7.7b) Find a function u that is harmonic on the disk $x^2 + y^2 < 6$ and that satisfies the boundary condition $u(x,y) = y + y^2$ on the disk's boundary. Write your answer in terms of Cartesian coordinates.

FOR STUDENTS ENROLLED IN 92.545.

3. (Pinchover & Rubinstein, problem 7.3) Solve the following BVP for the reduced Helmholtz equation with $k = 1$. Hint: Look for a solution in the form $u(x,y) = \sum_{n=1}^{\infty} X_n(x) \sin(ny)$

$$\begin{aligned} \Delta u - u &= 0 && \text{on } 0 < x < \pi, 0 < y < \pi \\ u(0,y) &= 1 && 0 < y < \pi \\ u(\pi,y) &= 0 && 0 < y < \pi \\ u(x,0) &= 0 && 0 < x < \pi \\ u(x,\pi) &= 0 && 0 < x < \pi \end{aligned}$$