

Optional Homework Assignment # 7

Due April 22

PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work. You may work together, but everyone must turn in his/her own homework set.

This homework assignment is optional. If you submit it, I will use the score to replace a lower homework score.

1. a) Find the Fourier series for the sawtooth function given by $f(x) = x$ on the interval $-\pi \leq x \leq \pi$.
- b) To what value does the Fourier series converge at $x = \pi/2$?
- c) To what value does the Fourier series converge at $x = \pi$?

2. Use the Fourier Transform to solve the following IVP for the heat equation:

$$u_t = u_{xx} \text{ on } -\infty < x < \infty, \quad u(x, 0) = e^{-x^2/2}$$

Hints: $\mathcal{F}\{e^{-ax^2}\} = \sqrt{\frac{\pi}{a}}e^{-k^2/(4a)}$ and $\mathcal{F}^{-1}\{e^{-bk^2}\} = \frac{1}{2\sqrt{\pi b}}e^{-x^2/(4b)}$

Warning: *Mathematica* uses a slightly different definition of the Fourier Transform. If you use *Mathematica* for this problem, do not use the above 2 formulas; use *Mathematica* to find the transforms and inverse transforms you need. The *Mathematica* command for the Fourier Transform of $f(x)$ is `FourierTransform[f[x], x, k]`, and the command for the inverse transform of $F(k)$ is `InverseFourierTransform[F[k], k, x]`