92.445/545 Partial Differential Equations Spring 2013

Homework Assignment # 8 Solutions

PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work. You may work together, but everyone must turn in his/her own homework set.

1. Use the Maximum Principle to find the maximum and minimum values of the solution u of the following BVP:

$$\begin{aligned} \Delta u &= 0 \quad \text{on } D = \Big\{(x,y)|x^2+y^2<4\Big\} \\ u(x,y) &= e^x \quad \text{on } \partial D = \Big\{(x,y)|x^2+y^2=4\Big\} \end{aligned}$$

According to the Maximum Principle, the maximum and minimum values of u occur on the boundary ∂D. The smallest and largest values of x on ∂D are -2 and 2, and $u(x,y) = e^x$ on ∂D. Therefore, $\boxed{e^{-2} \text{ is the minimum value of } u \text{ and } e^2 \text{ is the maximum value of } u \text{ on } D \cup \partial D}$

2. Use the Mean Value Property to find $u(0,0)$, where u is the solution of the following BVP:

$$\begin{aligned} \Delta u &= 0 \quad \text{on } D = \Big\{(x,y)|x^2+y^2<1\Big\} \\ u(x,y) &= 1-x^2 \quad \text{on } \partial D = \Big\{(x,y)|x^2+y^2=1\Big\} \end{aligned}$$

By the Mean Value Property, $u(0,0)$ equals the average value of u on ∂D:

$$u(0,0) = \frac{1}{2\pi}\int_0^{2\pi} u(\cos(\theta),\sin(\theta))\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \Big[1-\cos^2(\theta)\Big]\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \sin^2(\theta)\, d\theta =$$

$$\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{1}{2}-\frac{1}{2}\cos(2\theta)\right]\, d\theta \Rightarrow \boxed{u(0,0) = \frac{1}{2}}$$