
Finite Element Analysis for the Damage Detection of Light Pole Structures

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Outline

- Introduction
- Objective
- Literature Review
- Approach
- Finite Element Models
- Results and Discussion
- Proposed Methodology
- Conclusion
- Reference

Introduction

In December 2009, a 200-pound corroded light pole fell across the southeast expressway in Massachusetts.[1]



(Source: [1] I-team: Aging light poles a safety concern on mass. roads)

Introduction

1. Failures of light poles are **critical** as they are typically located adjacent to roadways, highway and bridges.
2. Failures of aging light poles can jeopardize the **safety of users** and damage adjacent structures. (e.g., residential houses, and electricity boxes.)



(Source: Internet)



(Source: Internet)

Introduction

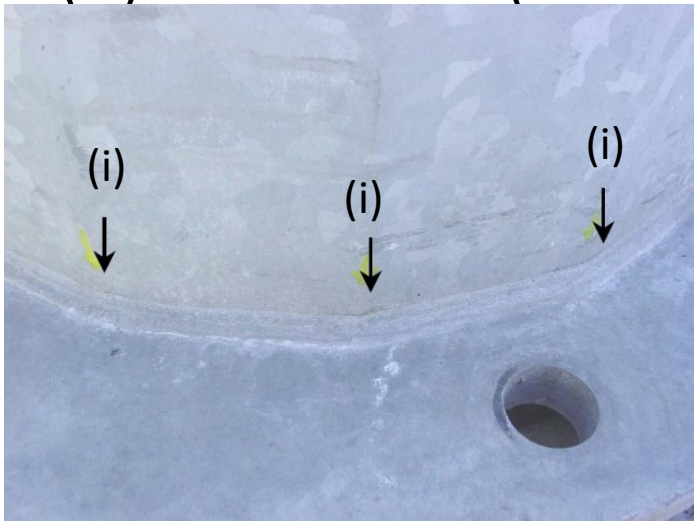
Therefore, aging light poles need to be repaired or removed before residents get hurt.

Objective

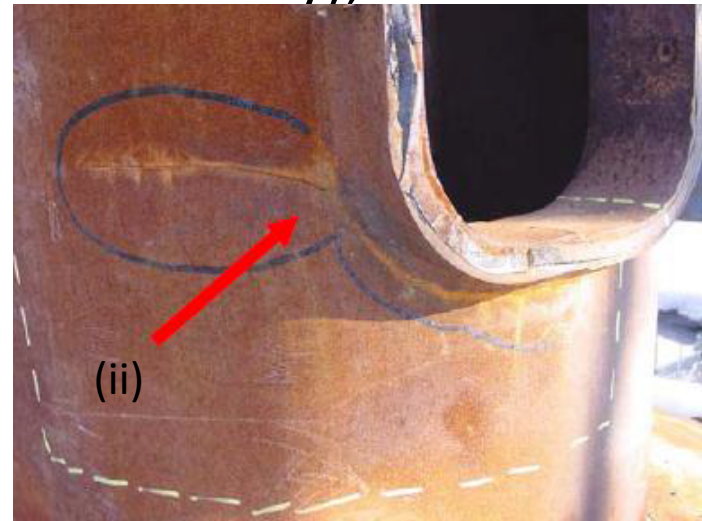
To develop a damage detection methodology for light poles structures.

Literature Review

1. There are three most common/possible damage locations in light poles (Garlich and Thorkildsen (2005) [14] , Caracoglia and Jones (2004)[7]; Conner et. al. (2005)[6])
 - (i) pole-to-baseplate connection,
 - (ii) handhole detail, and
 - (iii) anchor bolts (not considered in this study);



Cracks at bottom of the pole



Cracks at handhole detail

Literature Review

2. Changes in modal frequencies and mode shapes are expected while introducing damages into structures. (Lee and Chung (2000) [44]; Abdo and Hori (2002)[4])

For example, first mode modal frequency of a single degree of freedom (SDoF) system can be determined by following equation:

$$\omega = \sqrt{k/m}$$

where ω is first mode modal frequency, k is stiffness of the structure, and m is mass of structure.

Since introduction of damage reduces k (stiffness) of the structure. As a result,

$$k \downarrow \longrightarrow \omega \downarrow$$

Literature Review

3. Structural damages in light poles can be simulated by reducing local materials' properties (i.e. Young's modulus) in FE models. (Yan *et. al.* (2006) [42])

Since stiffness of a single degree of freedom (SDoF) system can be written as:

$$k=3EI/h^3$$

where E is Young's modulus of material, and I is moment of inertia, and h is the height of this SDoF.


$$E \longrightarrow k$$

Literature Review

4. Experimentally capture dynamic characteristics (such as modal frequencies and mode shapes) of light poles are difficult and time consuming. (Yan *et. al.* (2007)[43])

Literature Review

- How do damages effect light poles?
There will be changes in modal frequencies and mode shapes.
- Where are those effects? How much?
Three common damage locations. Use numerical methods to find out.
- Can those effects be represented by equations? (be quantified?)
Use numerical methods to find out.

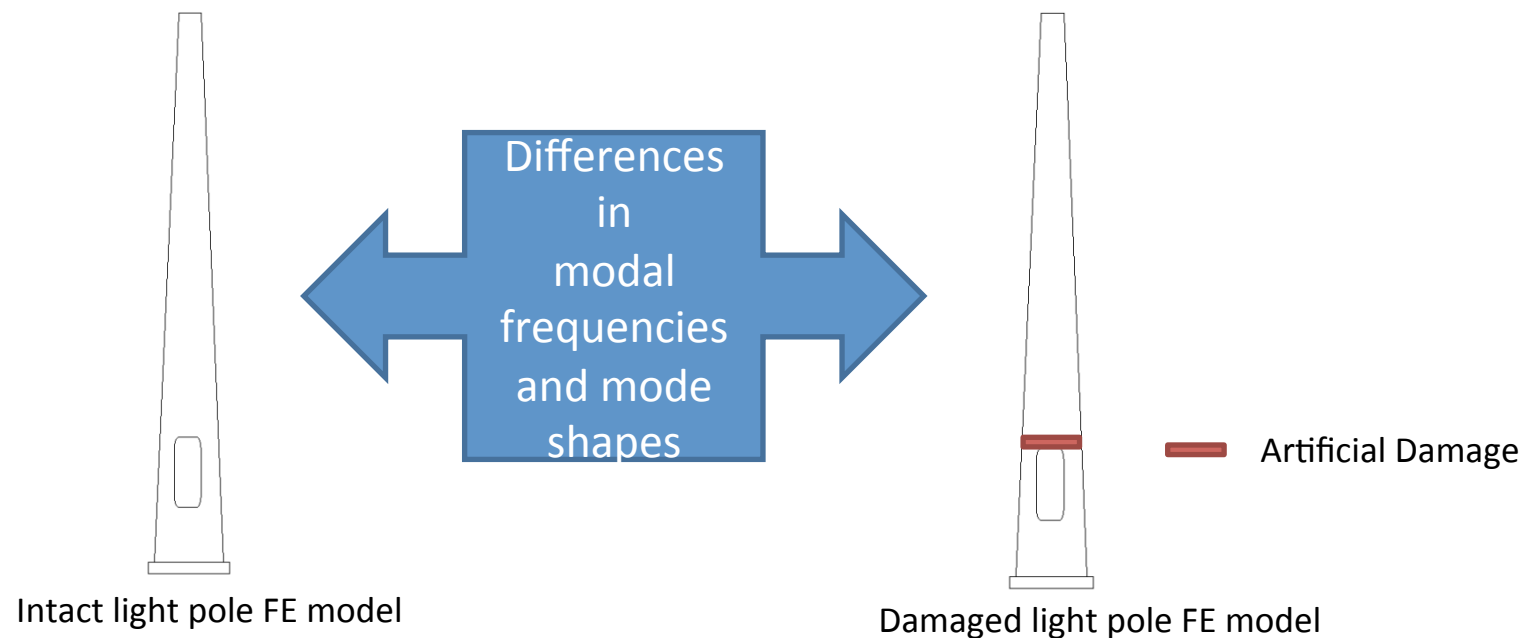
Approach

A research approach is determined based on:

1. Assume damages only occur at three most common damage locations.
2. Using dynamic responses (i.e., modal frequency and mode shapes) as parameters for investigating differences between intact and damaged light poles.
3. Using numerical methods (i.e., finite element method) instead of experimental methods.

Approach

- Simulate intact and damaged light poles by Finite Element (FE) method, and then study the differences in modal frequencies and mode shapes among intact and artificially damaged FE models.

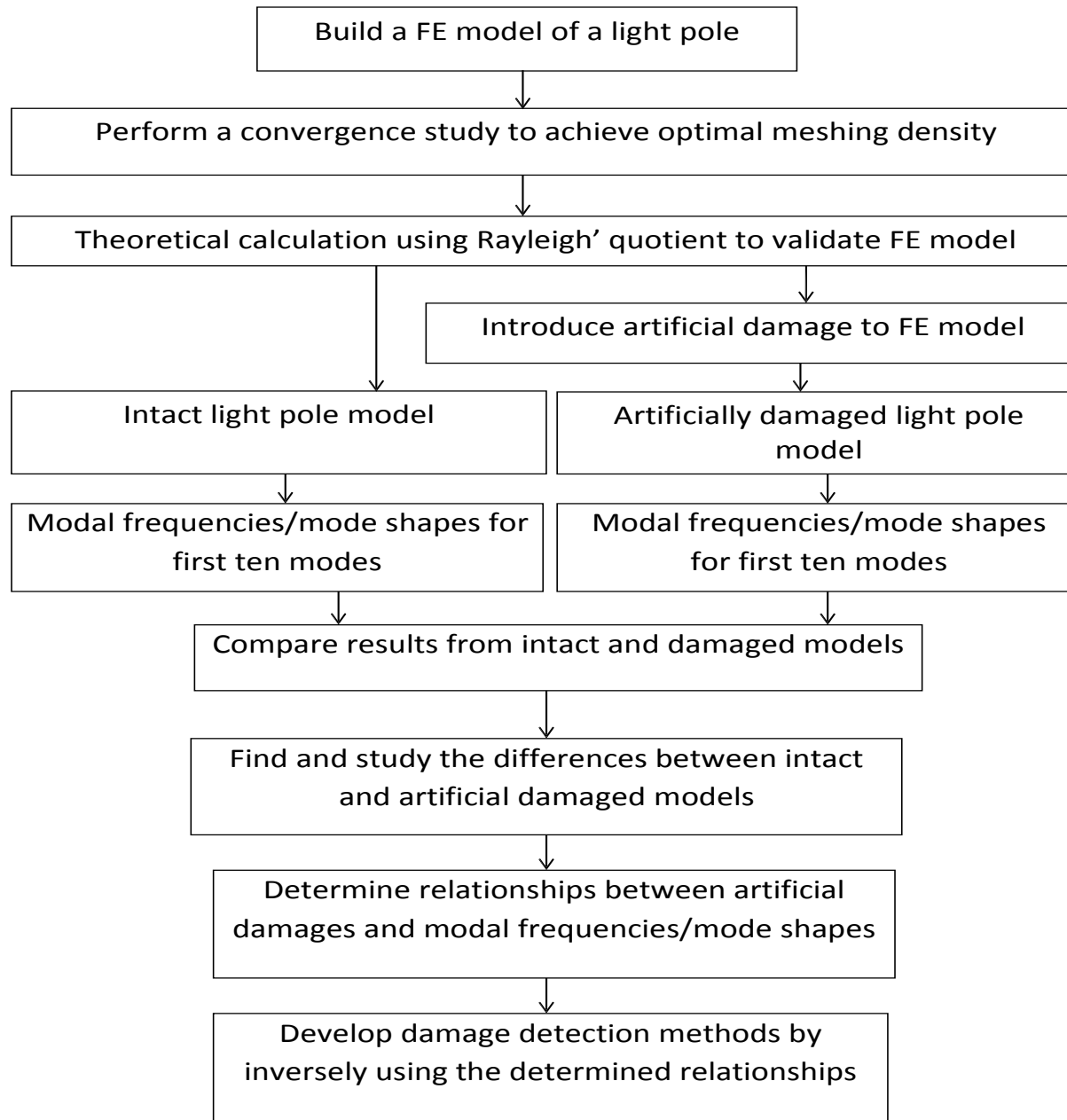


Approach

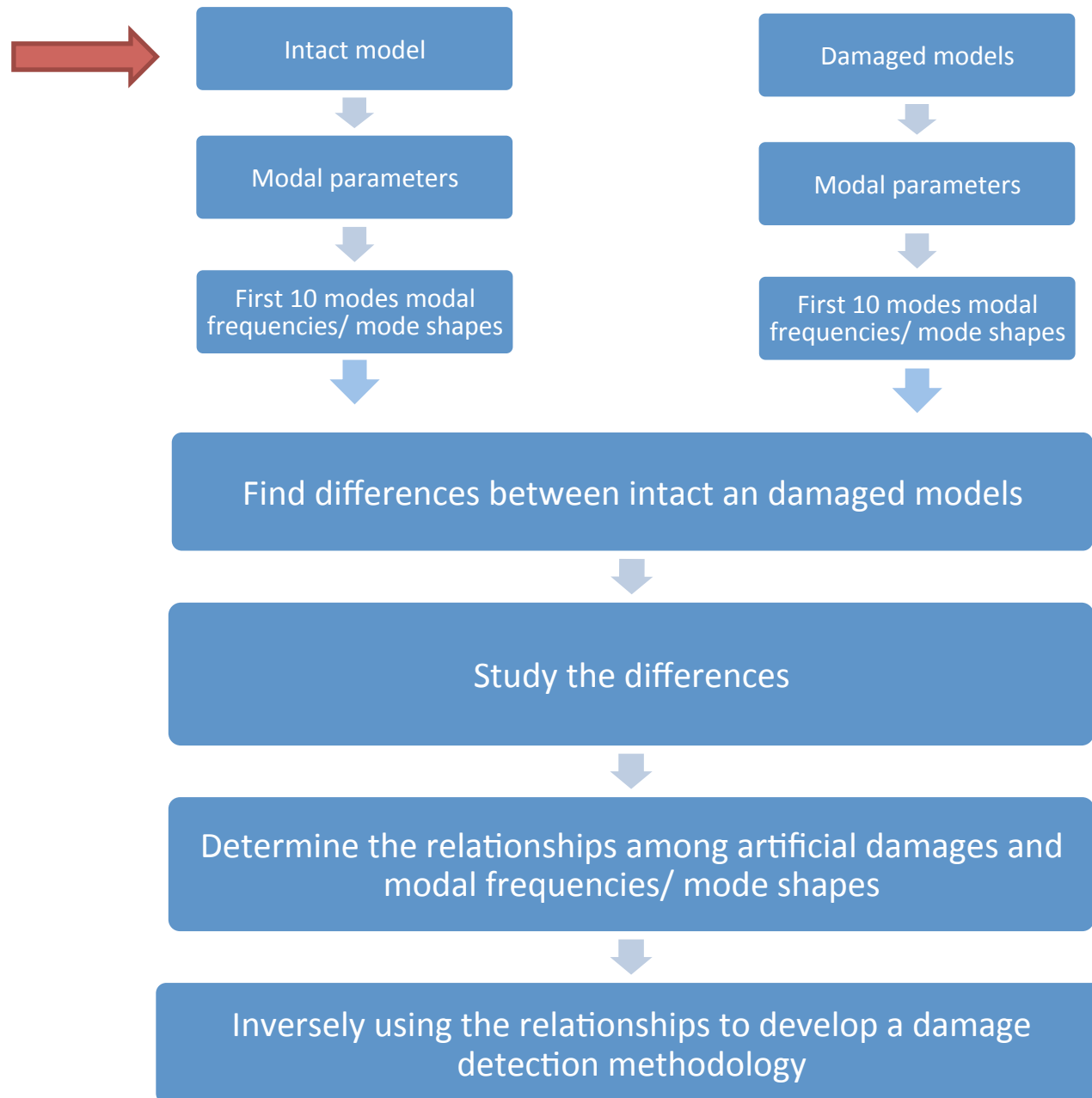
Assumptions for this research approach:

- First ten modes modal frequencies are available.
- FE light pole is an undamped structure.
- Damages only occur at pole-to-baseplate connection, and handhole detail.
- There is only one damage in any artificially damaged light pole.

Research Methodology



Roadmap



Finite Element Models

Intact model

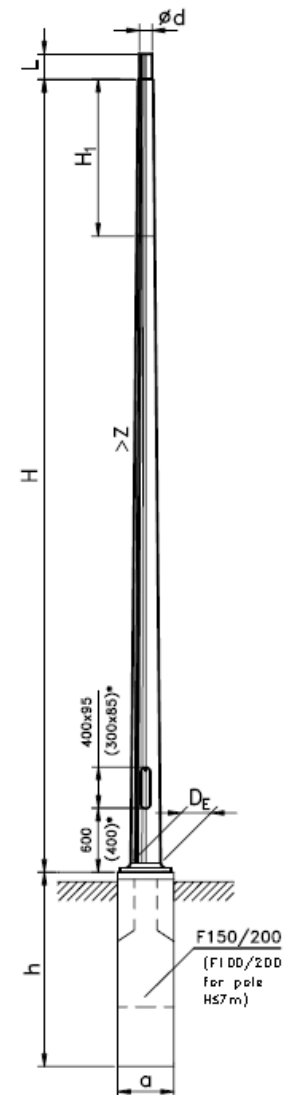
An FE model was created in ABAQUS® .

Finite Element Models

Configurations of an Example Light Pole



(source: ELEKTROMONTAŻ RZESZÓW SA: *Lighting poles and masts*, 2009)



*- dimensions for pole H \leq 7m

Finite Element Models

Technical data

TYPE	H	t _{bl}	H ₁	∅d/D _E	L	m	S	axaxh Type
	m	mm	m	mm	mm	kg	m ²	m
S-60SRwP/4	6		2,0	48; 60/140		68,0	1,47	0,3x0,3x1,0
S-70SRwP/4	7		2,0			79,0	1,71	F100/200
S-80SRwP/4	8		2,2			96,0	2,76	
S-90SRwP/4	9	4	2,5		100	104,0	3,41	
S-100SRwP/4	10		3,5	48; 60/170		110,0	3,65	0,3x0,3x1,5
S-110SRwP/4	11		2,2			128,0	3,89	F150/200
S-120SRwP/4	12		3,2			135,0	4,22	

Note: H₁ – reduction piece for straight pole is ordered as separate element.

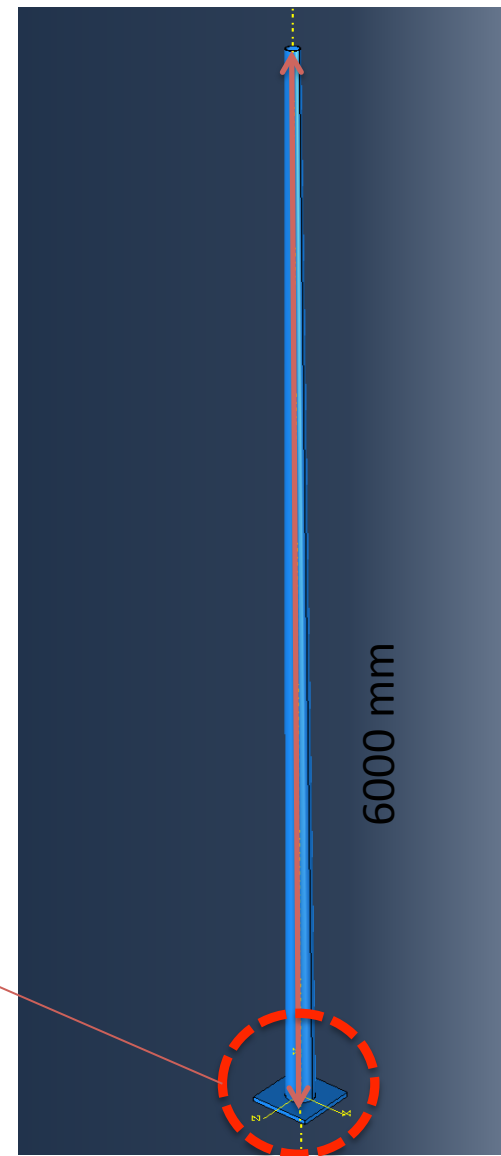
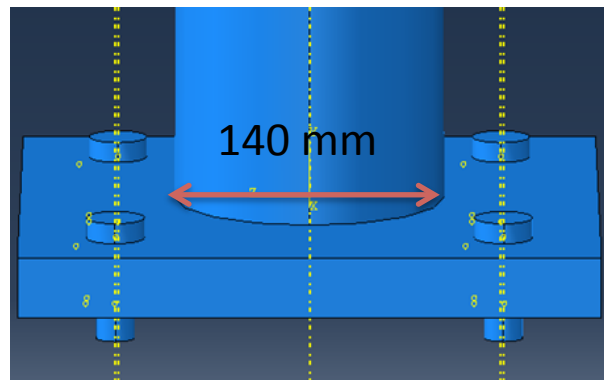
(source: ELEKTROMONTAŻ RZESZÓW SA: *Lighting poles and masts*, 2009)

-- Chosen geometry

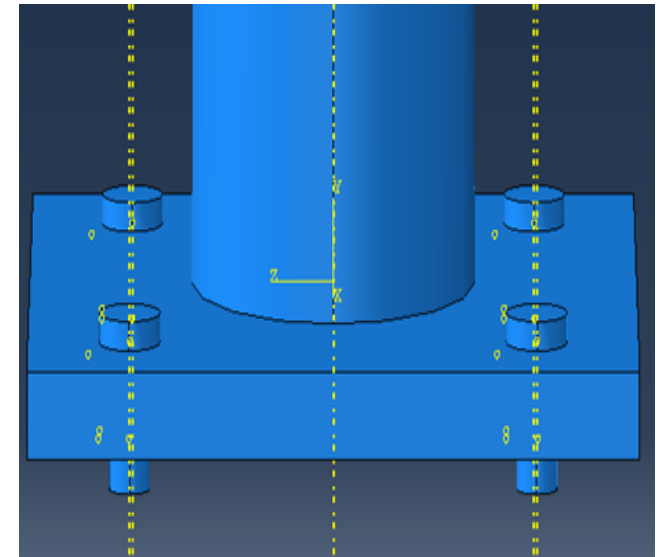
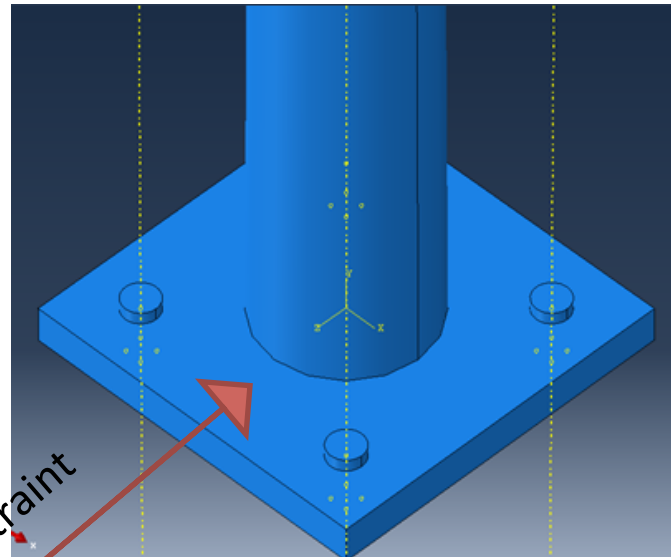
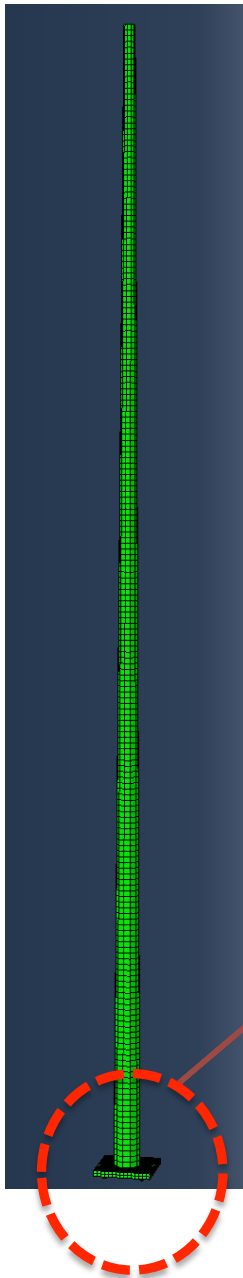
Finite Element Models

Materials (steel) & Geometries

Density:	7.85E-09 ton/mm ³
Young's Modulus:	207,000 MPa
Poisson's ratio:	0.3
Yield stress:	450 Mpa
Length of the pole:	6,000 mm
Diameter at top:	60 mm
Bottom:	140 mm



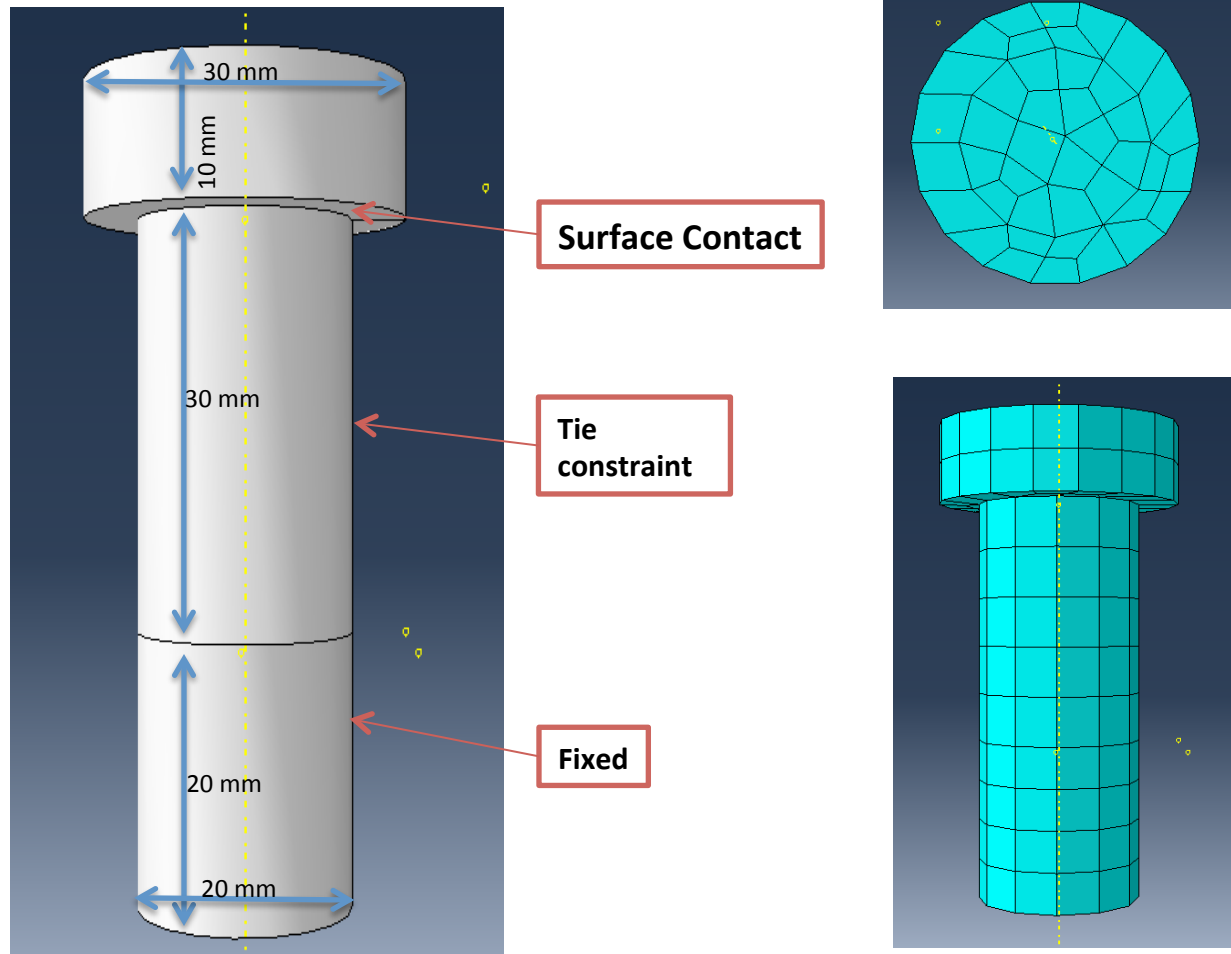
Base Plate Model



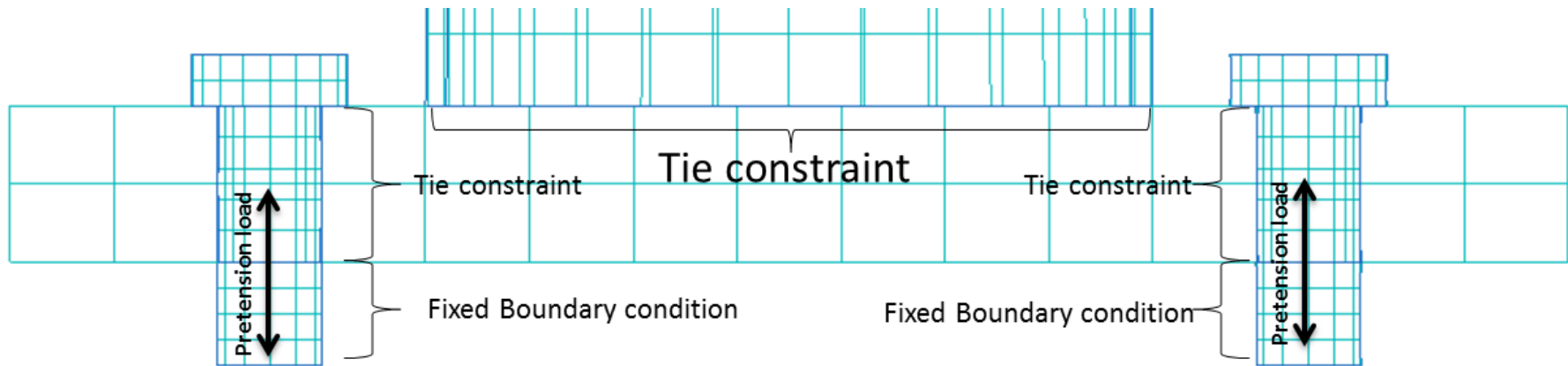
Tie constraint

- Base plate size: L x W x H: 300x300x30 (mm)
- Boundary condition: Tie constraint is used to assemble the pole and the base plate.

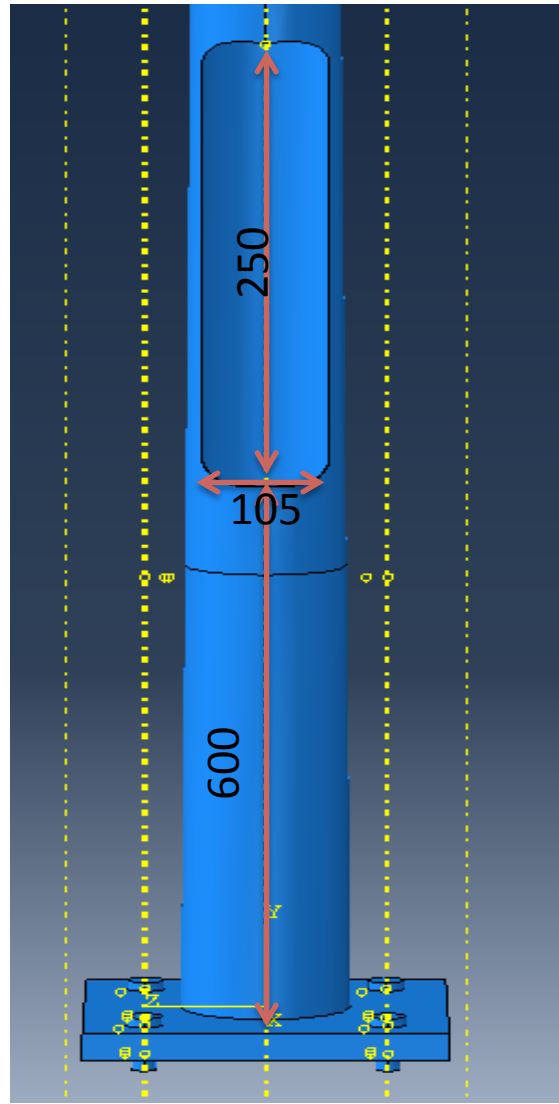
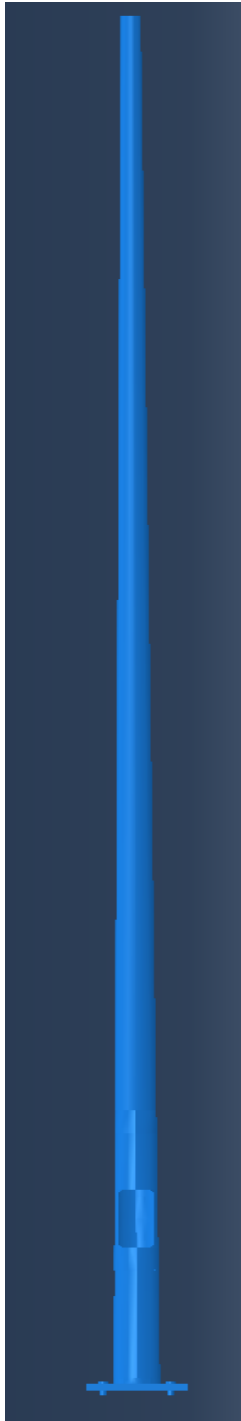
Bolt Models



Base Plate Model

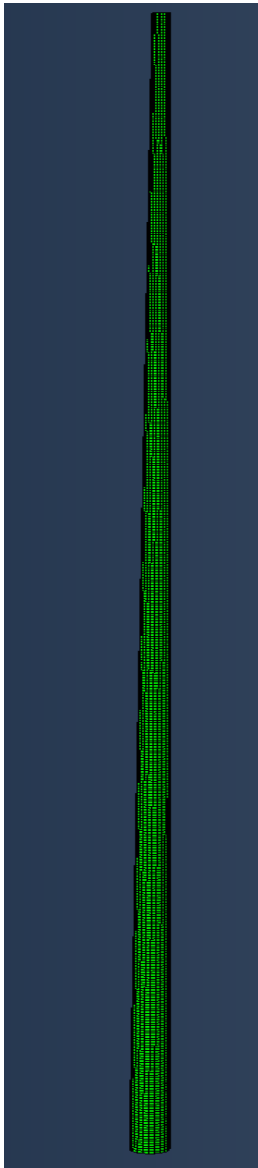


Handhole



- Handhole size:
250x105(mm)

Verification of an Intact FE Pole Model



The verification of a intact FE pole model is conducted by comparing first mode model frequencies between the FE result and a theoretical calculation result.

First mode model frequency of the FE pole model created in ABAQUS® is : **4.236 Hz (FE result)**.

0	Increment	0: Base State		
1	Mode	1: Value = 708.22	Freq = 4.2355	(cycles/time)
2	Mode	2: Value = 708.22	Freq = 4.2355	(cycles/time)
3	Mode	3: Value = 14192.	Freq = 18.960	(cycles/time)
4	Mode	4: Value = 14192.	Freq = 18.960	(cycles/time)
5	Mode	5: Value = 89791.	Freq = 47.691	(cycles/time)
6	Mode	6: Value = 89791.	Freq = 47.691	(cycles/time)

Theoretical Calculation

Given:

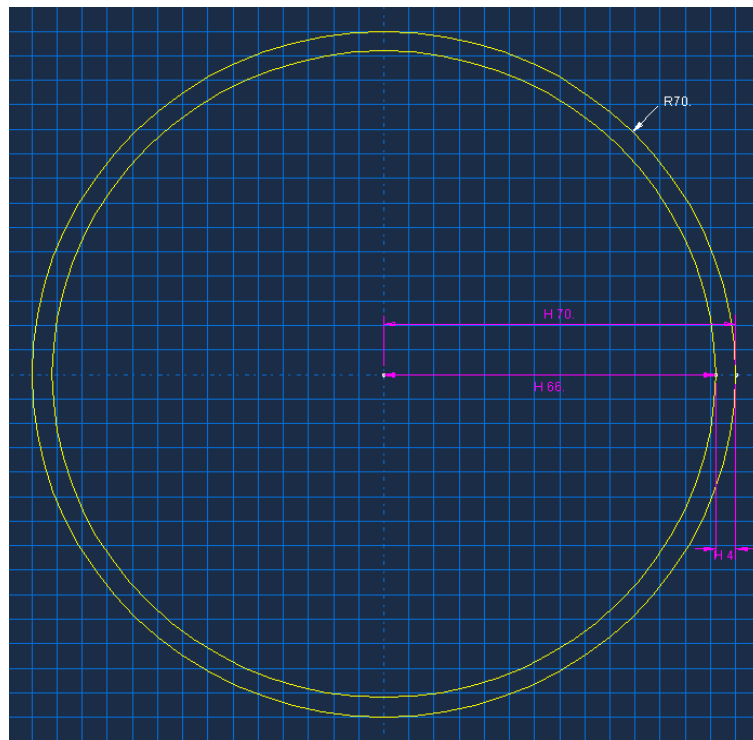
Density: 7.85E-09 Ton/mm²

Young's Modulus: 207000 MPa

Poisson's ratio: 0.3

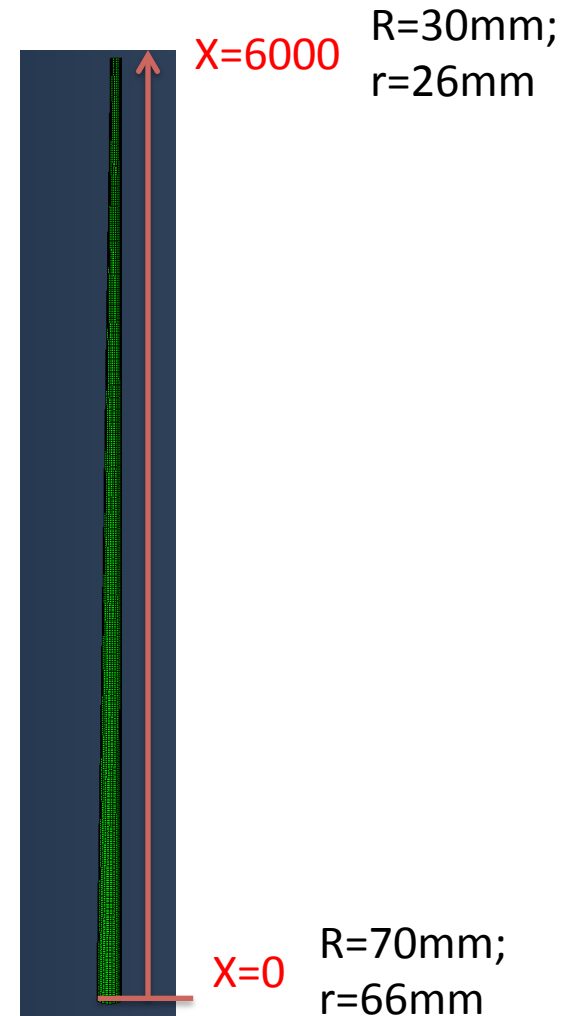
Yield strength: 450 Mpa

Length of the pole: 6000 mm



Cross section at base of pole(x=0)

R=70mm;
r=66mm



R=70mm;
r=66mm

Theoretically Computed Fundamental Frequency

1. Radius functions

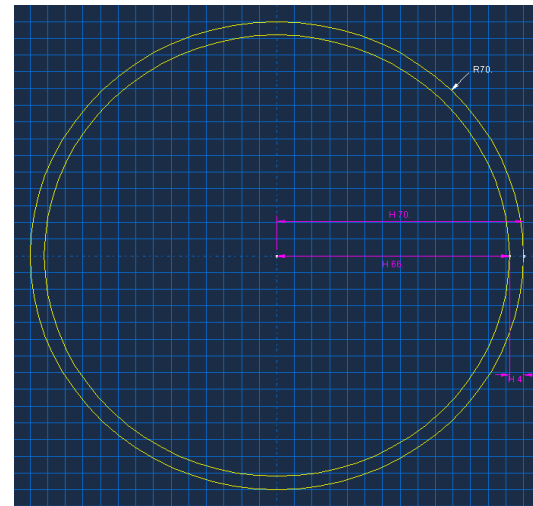
Radius of external edge: $\underline{R(x)} := \frac{1}{-150} \cdot x + 70$

Radius of internal edge: $r(x) := R(x) - 4$

2. Functions for mass and moment of inertia

$$\underline{m(x)} := \pi \cdot (R(x)^2 - r(x)^2) \cdot 7.85 \cdot 10^{-9}$$

$$I(x) := \left(R(x)^4 - r(x)^4 \right) \cdot \frac{\pi}{4}$$



Theoretically Computed Fundamental Frequency

3. Generalized mass, and generalized stiffness

$$m' := \int_0^L m(x) \psi(x)^2 dx$$

$$k' := \int_0^L E \cdot I(x) \cdot \left(\frac{d^2 \psi(x)}{dx^2} \right)^2 dx$$

Eq. 8.3.12

Anil K Chopra. *Dynamics of structures-Theory and applications to earthquake engineering*. Pg.312

where $\psi(x)$ is shape function of cantilever beams. The best-fit shape function is the one which provides lowest value of first mode modal frequency.

Theoretically Computed Fundamental Frequency

First mode modal frequency: $\omega := \left(\frac{k'}{m'} \right)^{0.5}$

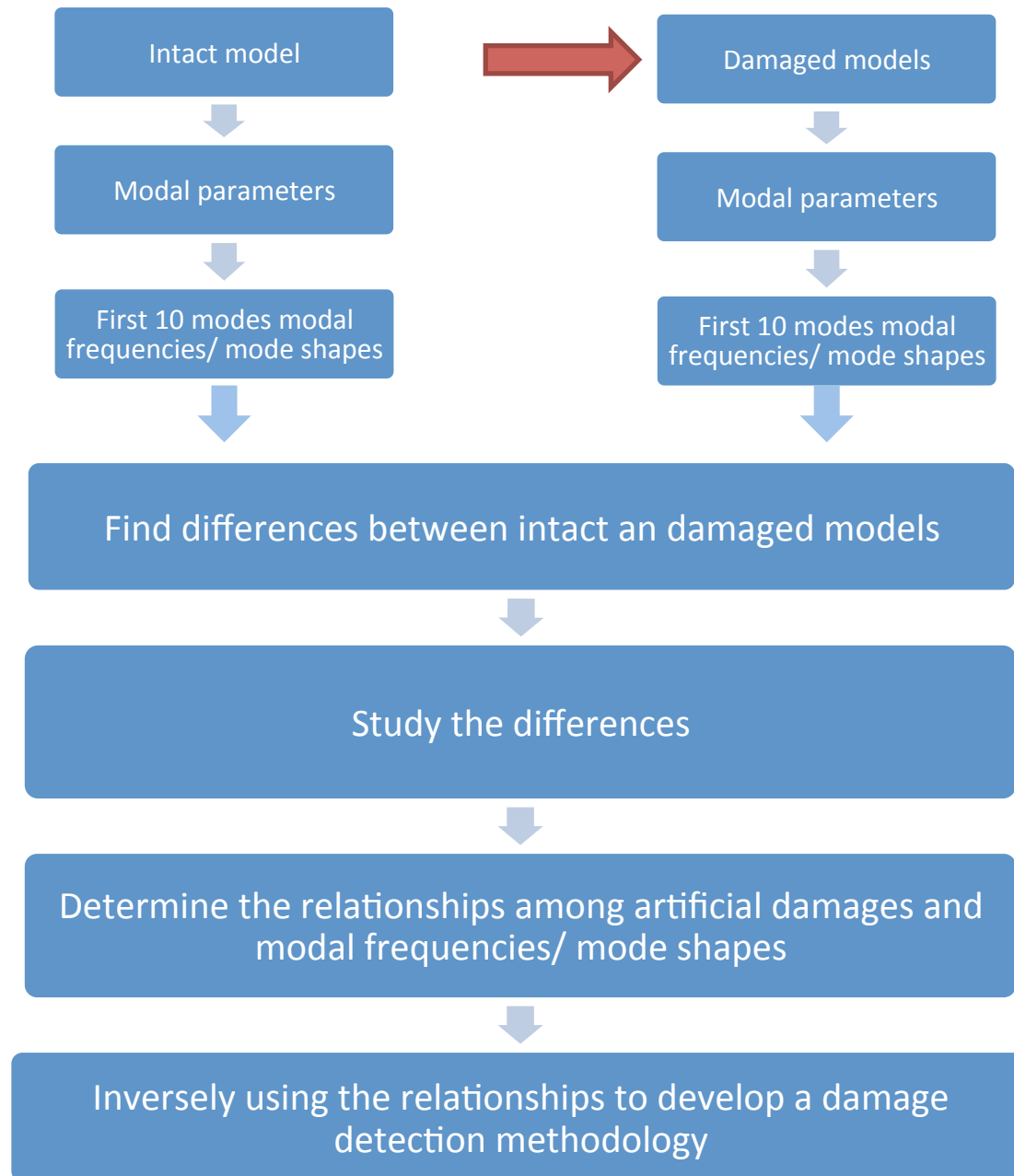
Shape function	$\psi(x) := \frac{3x^2}{2 \cdot 6000^2} - \frac{x^3}{2 \cdot 6000^3}$	$\psi(x) := 1 - \cos\left(\frac{\pi \cdot x}{2 \cdot 6000}\right)$	$\psi(x) := \frac{x^2}{6000^2}$
f (Hz)	4.361	4.298	4.396

Theoretical result: lowest value of fundamental frequency is **4.298 Hz** ();

FE result: **4.236 Hz**.

Only 1.4% of difference. This means the FE result is correct and accurate.

Roadmap



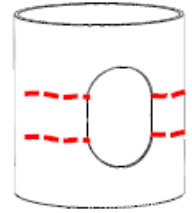
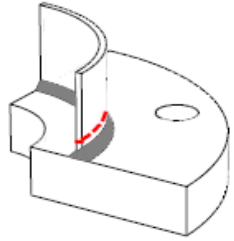
Finite Element Models

Damaged models

Damaged models were simulated by introducing artificial damages to intact light pole models.

Damage Simulations

Most common damage location:

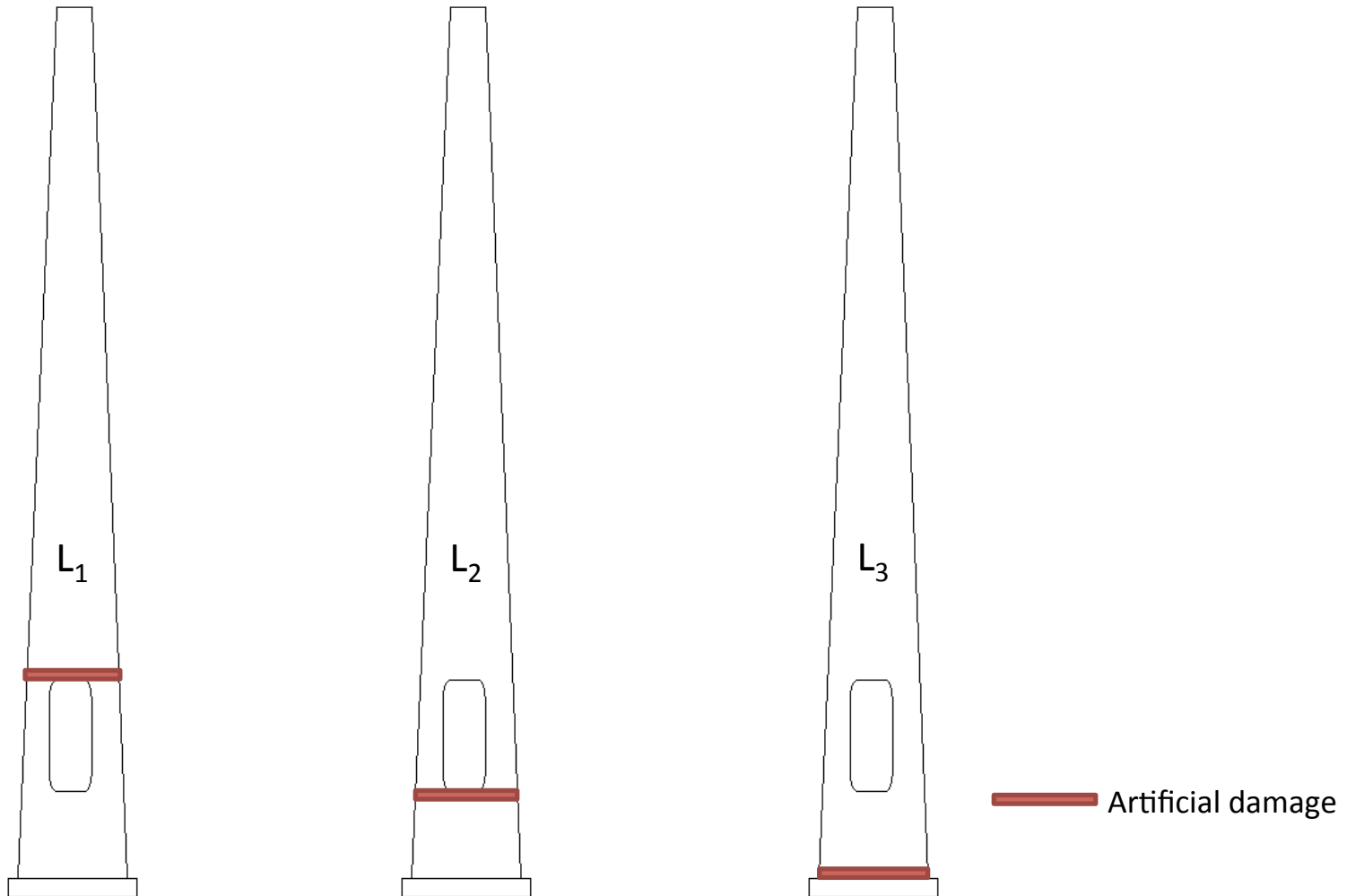
Description	Finite Life Constant, $A \times 10^8$ (ksi ³ (MPa ³))	Threshold, $(\Delta F)_{TH}$ (ksi (MPa))	Potential Crack Location	Illustrative Example
SECTION 3 — HOLES AND CUTOUTS				
3.1 Net section of un-reinforced holes and cutouts.	250.0 (85200)	24.0 (165)	In tube wall at edge of unreinforced handhole.	
4.6 Full penetration groove-welded tube-to-transverse plate connections welded from both sides with back-gouging (without backing ring).	$K_F \leq 1.6 : 11.0$ (3750) $1.6 < K_F \leq 2.3 : 3.9$ (1330)	$K_I \leq 3.2 : 10.0$ (69) $3.2 < K_I \leq 5.1 : 7.0$ (48) $5.1 < K_I \leq 7.2 : 4.5$ (31)	In tube wall at groove-weld toe.	

(Source: NCHRP Report, *Cost-Effective Connection Details for Highway Sign, Luminaire, and Traffic Signal Structures*)

Damage Simulations

1. Location:

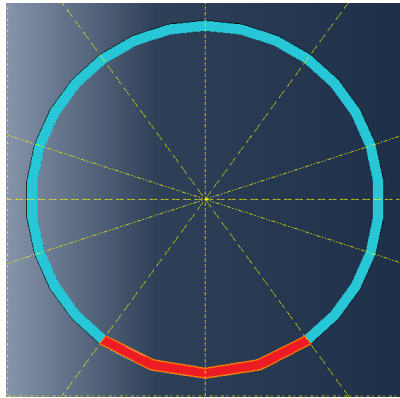
Artificial damages have three damage location: L_1 , L_2 and L_3 .



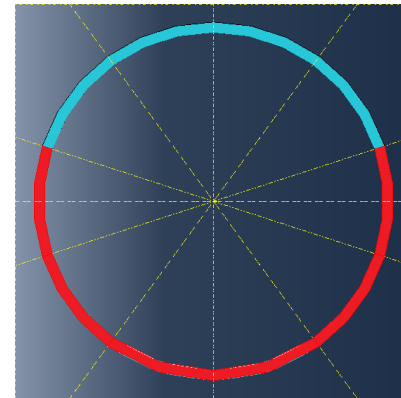
Damage Simulations

2. Different damage sizes (ΔA).

Artificial damages have **five** different sizes, including: $\Delta A \in [0.2A, 0.4A, 0.6A, 0.8A, 1.0A]$, where A is the total cross-sectional area.



0.2A – 20% Cross-section area damaged
(at location L3)



0.6A -- 60% Cross-section area damaged
(at location L3)

Damage Simulations

3. Damage levels (ΔE).

Damages are simulated by reducing Young's Modulus.

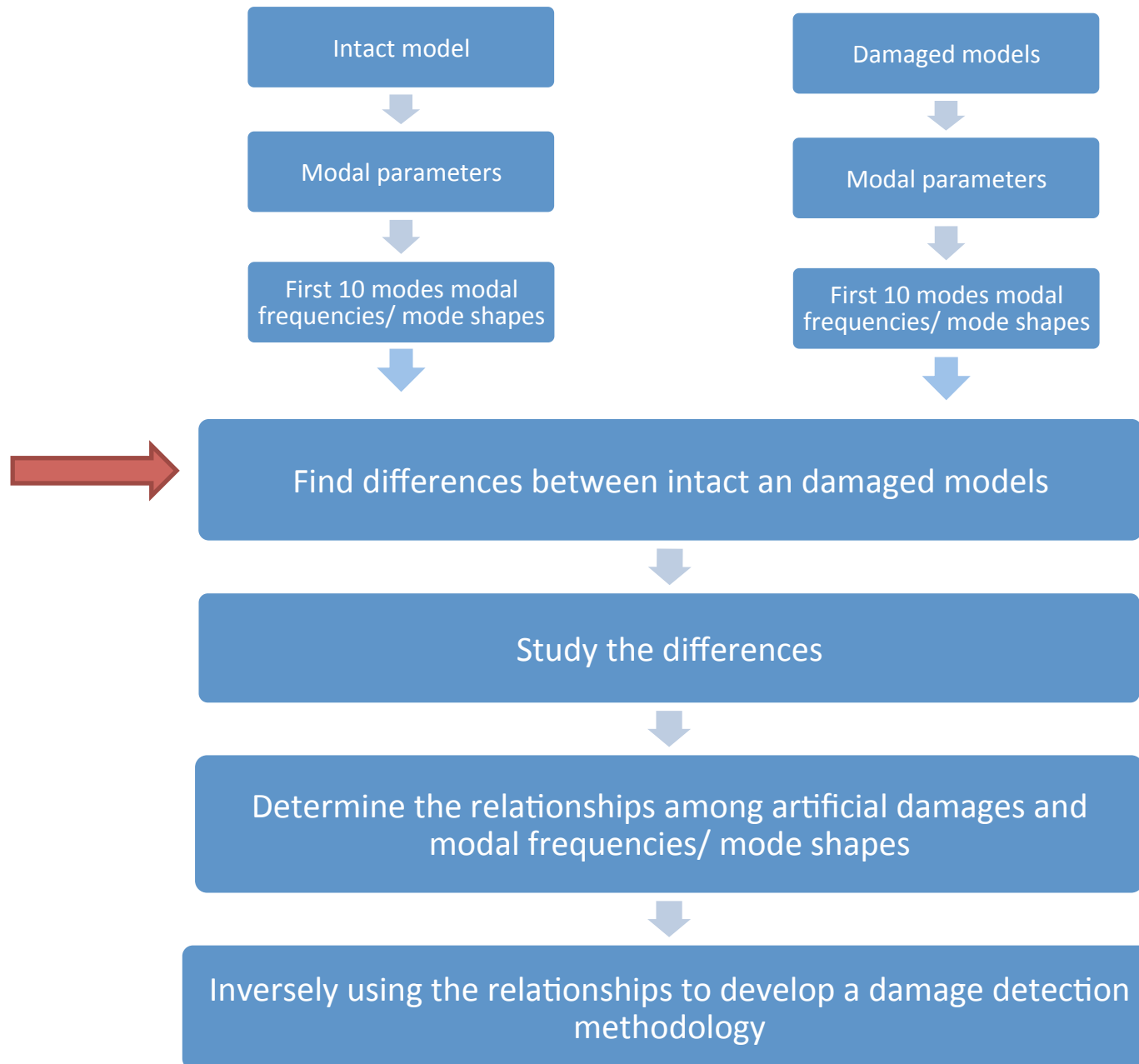
Including five levels: $\Delta E \in [0.1, 0.3, 0.5, 0.7, 0.9] * E$, where E is the Young's modulus of intact material.

Obtain first ten modes modal frequencies and mode shapes from different damaged models (listed in following) and intact model.

Group A														
$\Delta E=50\%$ of Youngs modulus														
$\Delta A=20\%$			$\Delta A=40\%$			$\Delta A=60\%$			$\Delta A=80\%$			$\Delta A=100\%$		
Scenario A-1			Scenario A-2			Scenario A-3			Scenario A-4			Scenario A-5		
L_1	L_2	L_3	L_1	L_2	L_3	L_1	L_2	L_3	L_1	L_2	L_3	L_1	L_2	L_3

Group B														
$\Delta A=100\%$ of total area														
$\Delta E=90\%$			$\Delta E=70\%$			$\Delta E=50\%$			$\Delta E=30\%$			$\Delta E=10\%$		
Scenario B-1			Scenario B-2			Scenario B-3			Scenario B-4			Scenario B-5		
L_1	L_2	L_3	L_1	L_2	L_3	L_1	L_2	L_3	L_1	L_2	L_3	L_1	L_2	L_3

Roadmap



Results and Discussion

Definition:

When the intact light pole is **known**, **modal frequency difference** can be computed by the following equation:

$$\Delta f_i^j = \frac{(f_i^j|_{intact} - f_i^j|_{damaged})}{f_i^j|_{intact}} * 100\%$$

When Δf_i^j is the modal frequency difference of a damaged pole in the i th mode with a damage located at L_j , $f_i^j|_{intact}$ is the modal frequency of the intact model, and $f_i^j|_{damaged}$ is the modal frequency of a damaged model.

Results and Discussion

Definition:

- Sensitive modes:

Out of first ten modes, the modes whose modal frequency differences **exceed** the defined threshold value t_s .

Threshold value t_s : 1.25 times the average modal frequency differences of the first ten modes.

$$t_s = 1.25 \sum_{i=1}^{10} f_{ij} / 10$$

Results and Discussion

Definition:

- Insensitive modes:

Out of first ten modes, the modes whose modal frequency differences **lower** than the defined threshold value t_i .

Threshold value t_i : 0.25 times the average modal frequency differences of the first ten modes.

$$t_s = 0.25 \sum_{i=1}^{10} f_{ij} / 10$$

Results and Discussion

Curvature of mode shapes can be computed by Central Difference Equation:

$$\phi''(x)_n = \frac{\phi(x)_{n+1} - 2\phi(x)_n + \phi(x)_{n-1}}{d^2}$$

where $\phi(x)$ is the displacement of mode shape at node n , d is displacement between two nodes, and $\phi''(x)$ is the curvature of mode shape at node n .

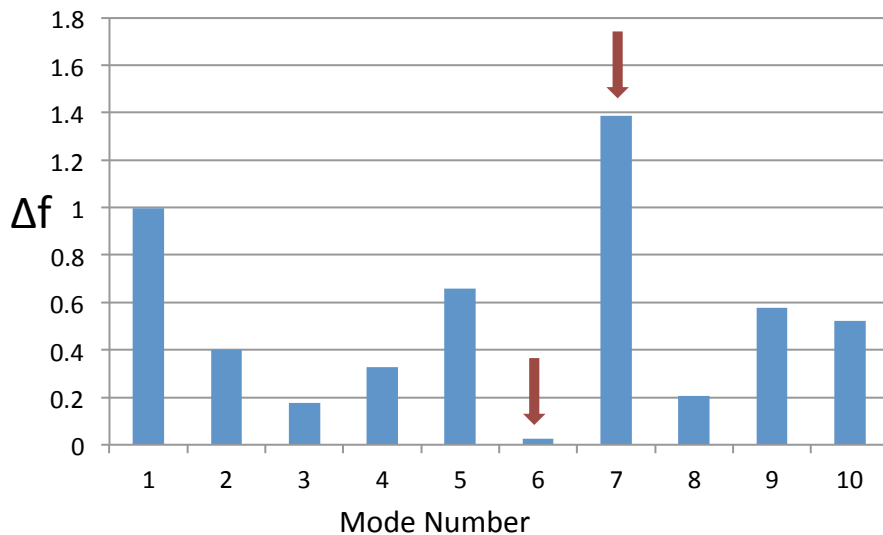
Changes in curvature of mode shapes can be computed by the following equation:

$$\Delta r_{\phi_n''} = \frac{\phi_n''|_{\text{damaged}}}{\phi_n''|_{\text{intact}}}$$

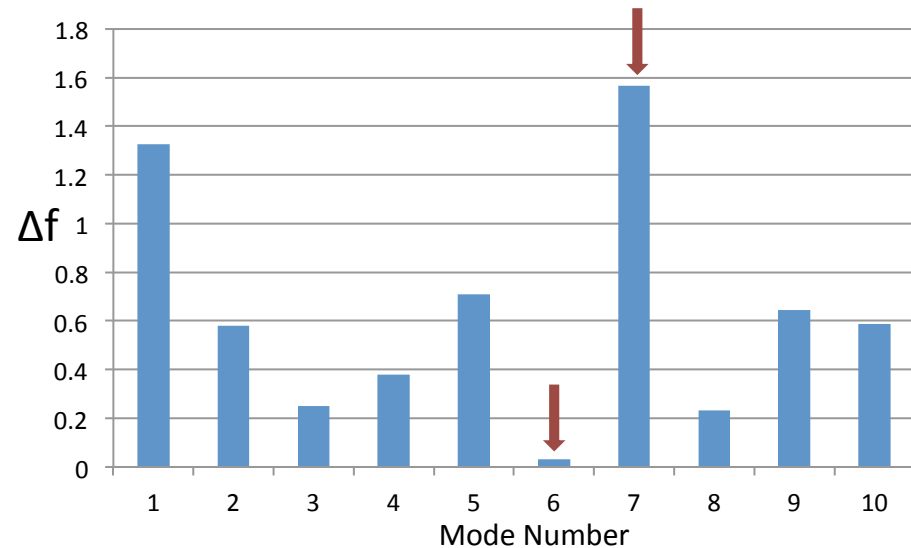
Summary of FE Results

Three patterns were found from FE results on modal frequencies.

1. In different damage scenarios with same damage location, some modes always have highest/lowest value in modal frequency differences (Δf_i^j).



Damage location: L1
Damage size: 40% A
Damage level: 50% E



Damage location: L1
Damage size: 100% A
Damage level: 50% E

Summary of FE Results

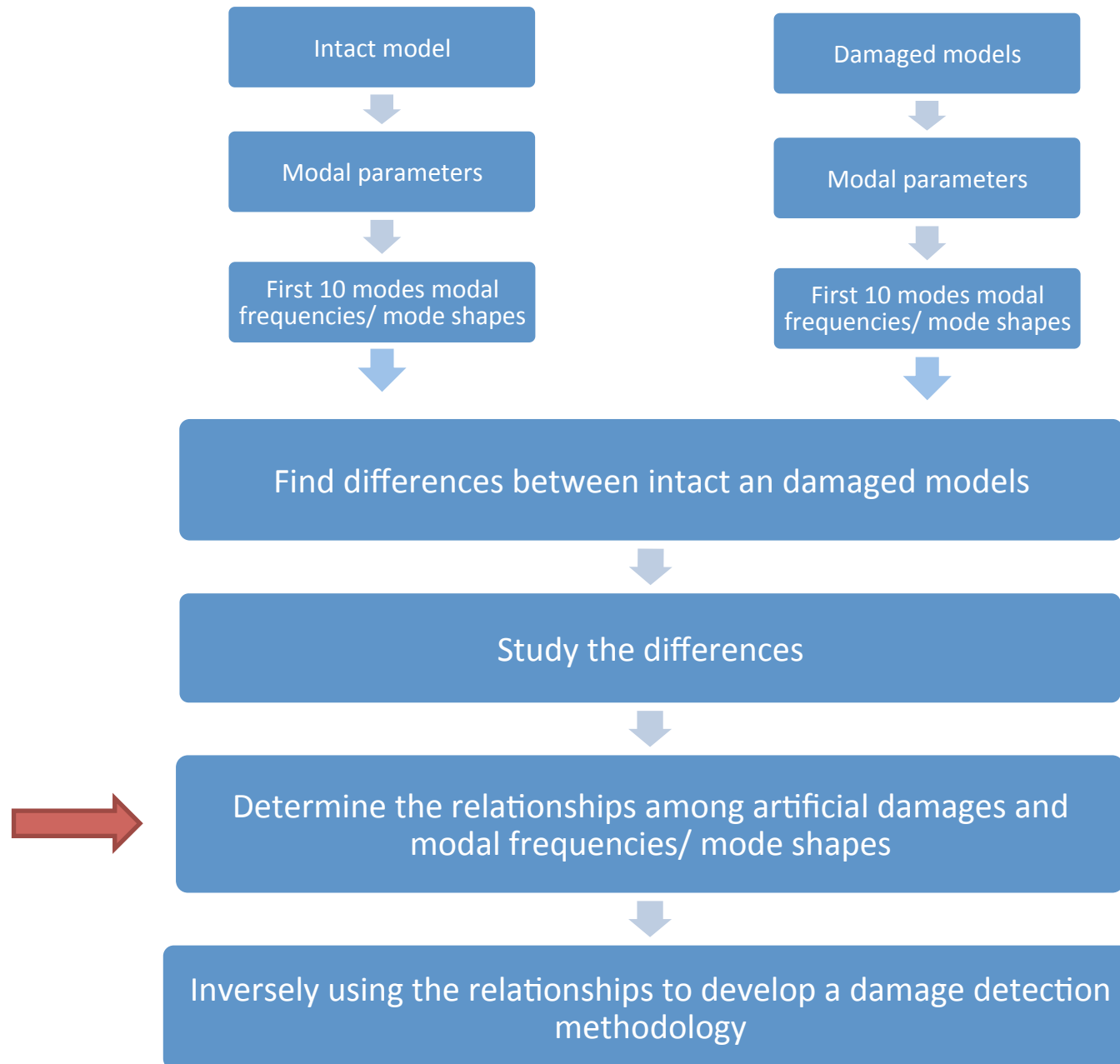
Sensitive/ insensitive modes for each damage location:

Table of sensitive/insensitive modes

Location	Sensitive modes	Insensitive modes
L_1	1, 7	6
L_2	1, 7	8, 10
L_3	9 or 10	7

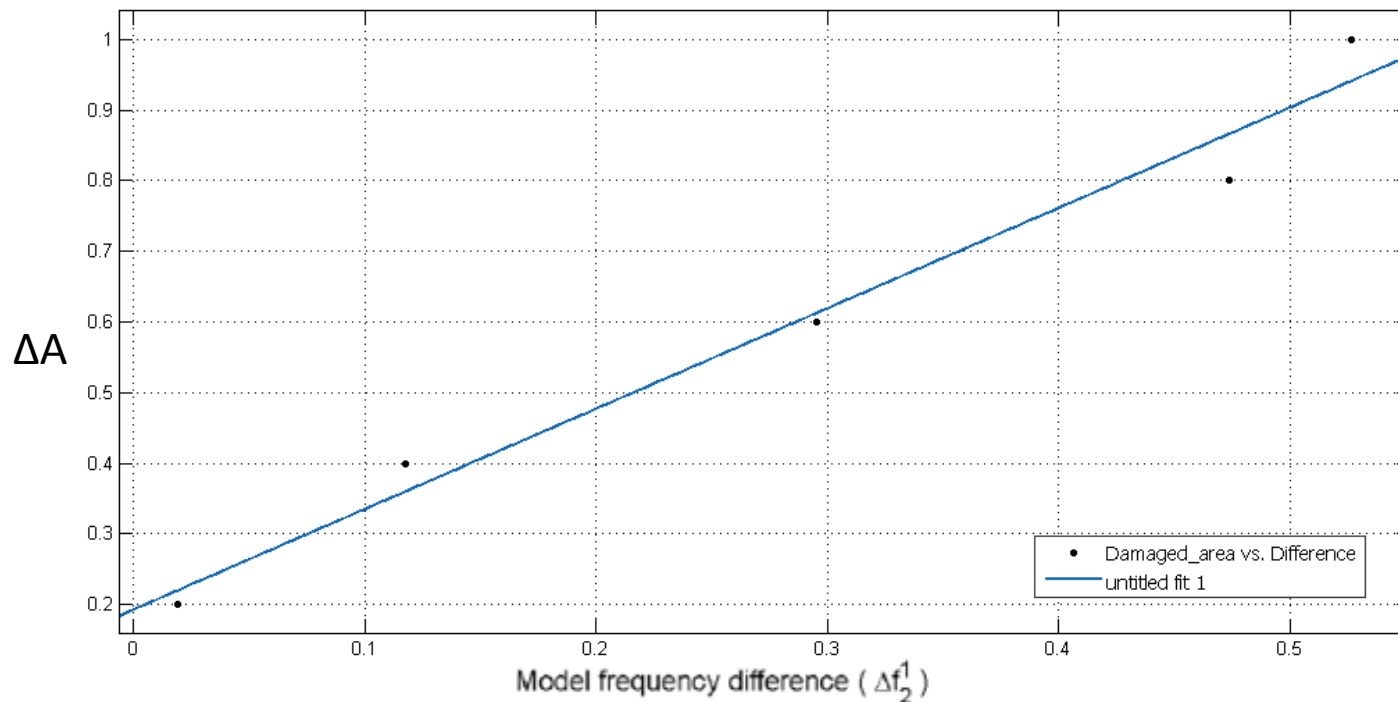
The combination of sensitive modes and insensitive modes is **unique for each damage location.**

Roadmap



Summary of FE Results

2.(1) Linear relationships were found between damage sizes and modal frequency differences.



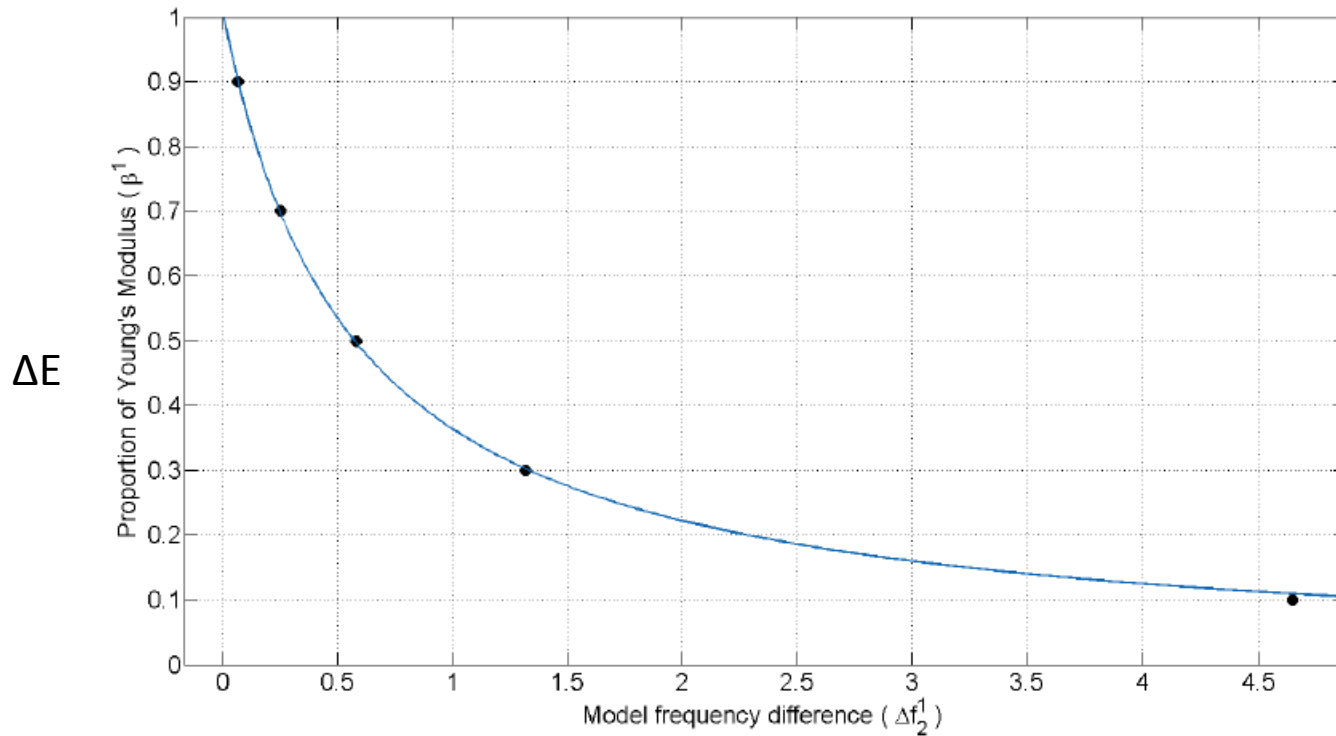
Quantification of Damages

Linear relationships can be described by the following equation:

Damage size $\alpha^j = a\Delta f_i^j + b$

Location(j)	Best-fit mode(i)	a	b	R^2
1	10	0.0255	0.4614	0.9841
2	6	0.0526	0.1665	0.9966
3	4	0.6858	-0.1251	0.9750

2. (2) Nonlinear relationships were found between damage levels (reduction in Young's modulus) and modal frequency differences.



Quantification of damage level:

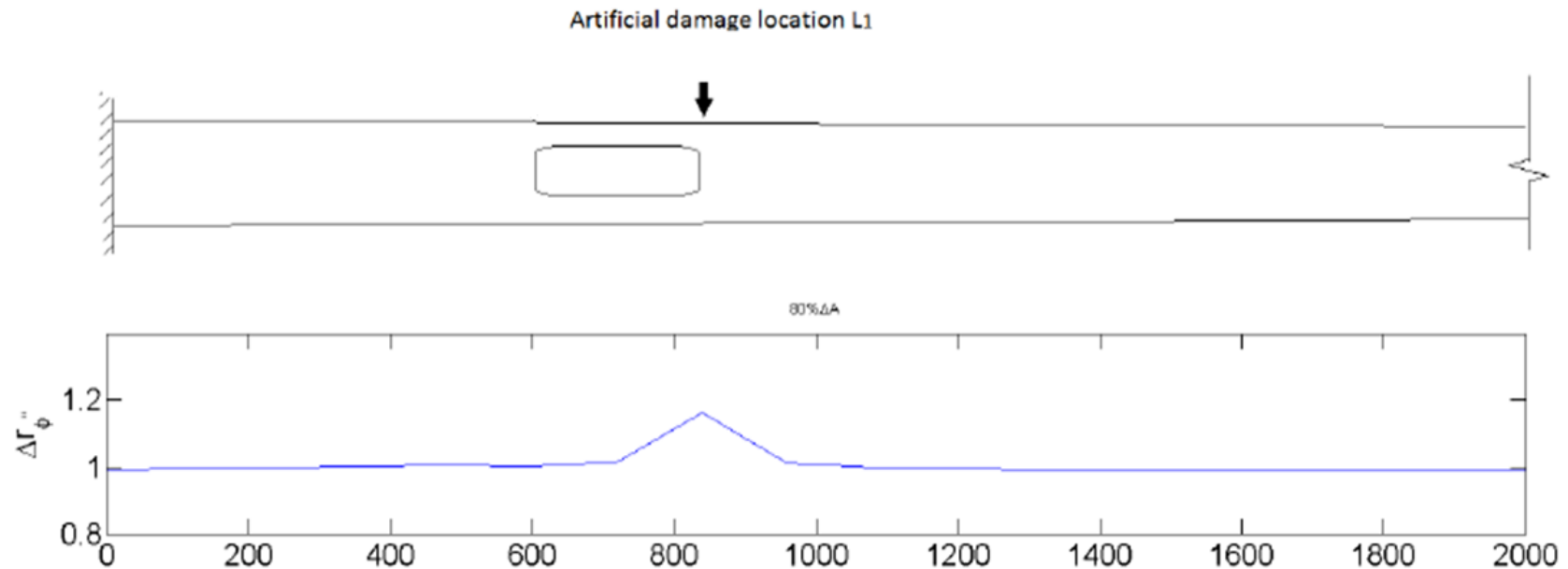
Relationship between damage level and modal frequency differences can be described by the following equation:

$$\text{Damage level } \beta^j = c \ln(\Delta f_i^j) + d$$

Location(j)	Best-fit mode(i)	c	d	R^2
1	2	-0.195	0.3900	0.9911
2	2	-0.194	0.3879	0.9914
3	8	-0.199	0.3628	0.9914

Summary of FE results

3. Curvatures of second mode shapes changes the most ($\Delta r_{\phi'' , \max}$) at damage location.



Special Case: Blind-test

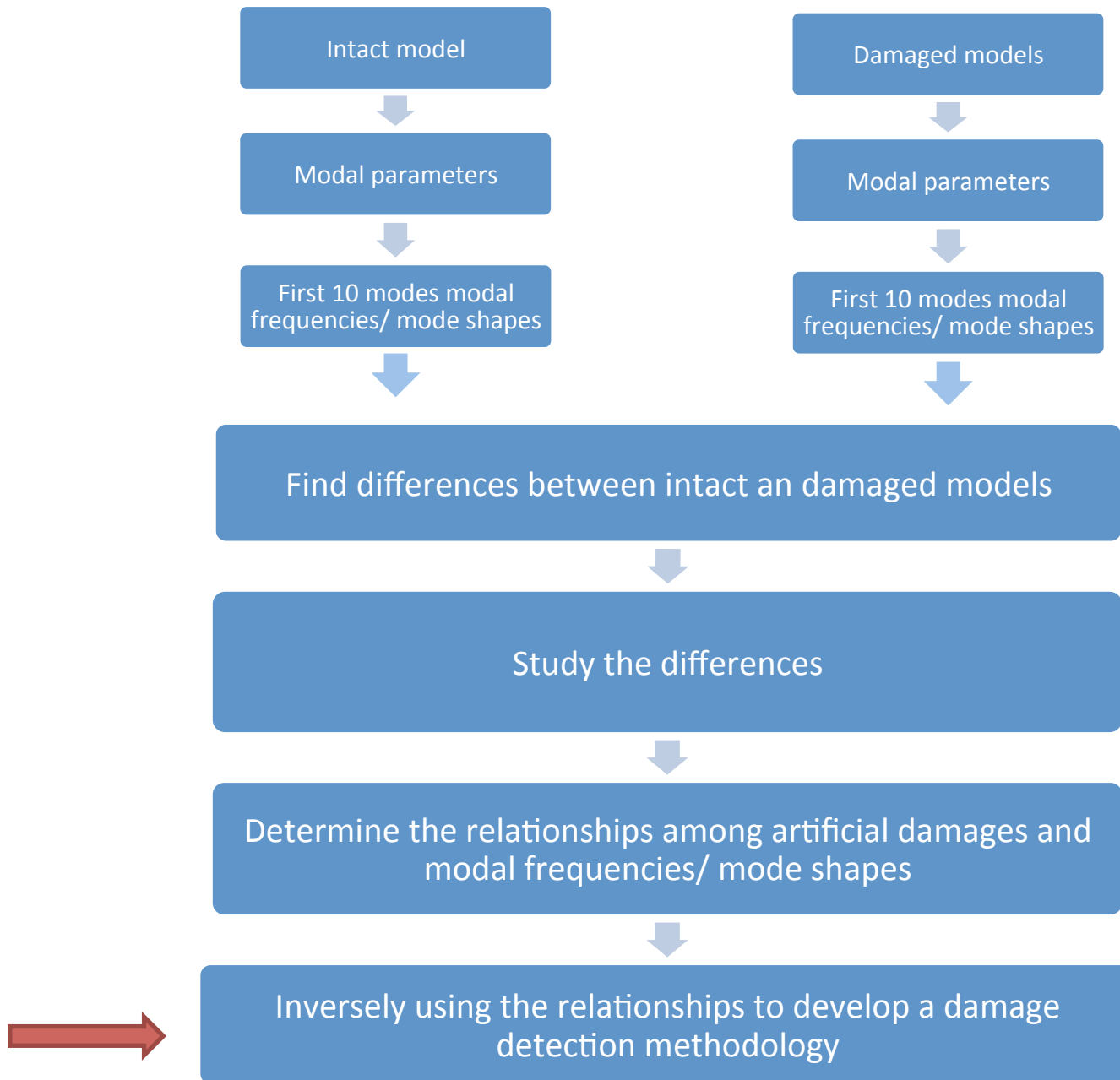
- Assumption: the intact light pole is unavailable.
- Modal frequencies of plural light poles can always be obtained.
 - 1) Pick an arbitrary light pole as baseline instead of intact light pole.
 - 2) Use adjusted equation to compute modal frequency difference.

$$\Delta f_i^j = \left| \frac{(f_i^j|_{baseline} - f_i^j|_{damaged})}{f_i^j|_{baseline}} \right| * 100\%$$

- 3) Determine the sensitive/insensitive modes using moving thresholds t_s and t_i .
- 4) Check the following Table and determine the damage location.

Location	Sensitive modes	Insensitive modes
L_1	1, 7	6
L_2	1, 7	8, 10
L_3	9 or 10	7

Roadmap



Proposed Methodology

1. Extract first 10 **model frequencies/ mode shapes** from an intact model & unknown models;
2. Compute the modal frequency differences and changes in mode shapes of the unknown light poles;
3. Compute thresholds t_s and t_i , and use them to determine sensitive/insensitive modes;
4. Locate the damage by checking the combination of sensitive and insensitive modes of unknown light poles in *Table of sensitive/insensitive modes*; or locate the damage by finding $\Delta r_{\phi'', \max}$;
5. Use obtained linear/non-linear equations to quantify the damage.

Conclusion

1. Locate damage using modal frequency -- since the combination of sensitive modes and insensitive modes is unique for each damage location, one can locate the damage of light pole by checking the following table:

Table of sensitive/insensitive modes

Location	Sensitive modes	Insensitive modes
L_1	1, 7	6
L_2	1, 7	8, 10
L_3	9 or 10	7

Conclusion

2. Quantify damage -- substituting modal frequency difference into following equations:

Damage size: $\alpha^j = a\Delta f_i^j + b$

Damage level: $\beta^j = c \ln(\Delta f_i^j) + d$

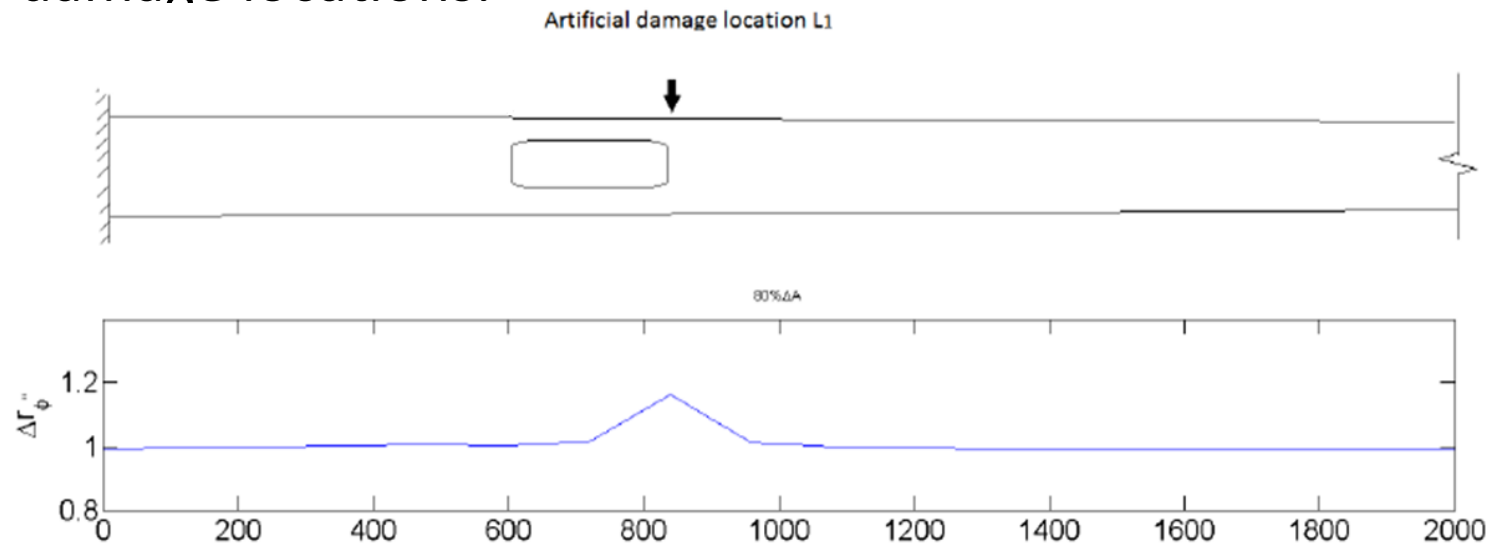
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1	10	0.0255	0.4614	0.9841
2	6	0.0526	0.1665	0.9966
3	4	0.6858	-0.1251	0.9750

Location(j)	Best-fit mode(i)	c	d	R^2
1	2	-0.195	0.3900	0.9911
2	2	-0.194	0.3879	0.9914
3	8	-0.199	0.3628	0.9914

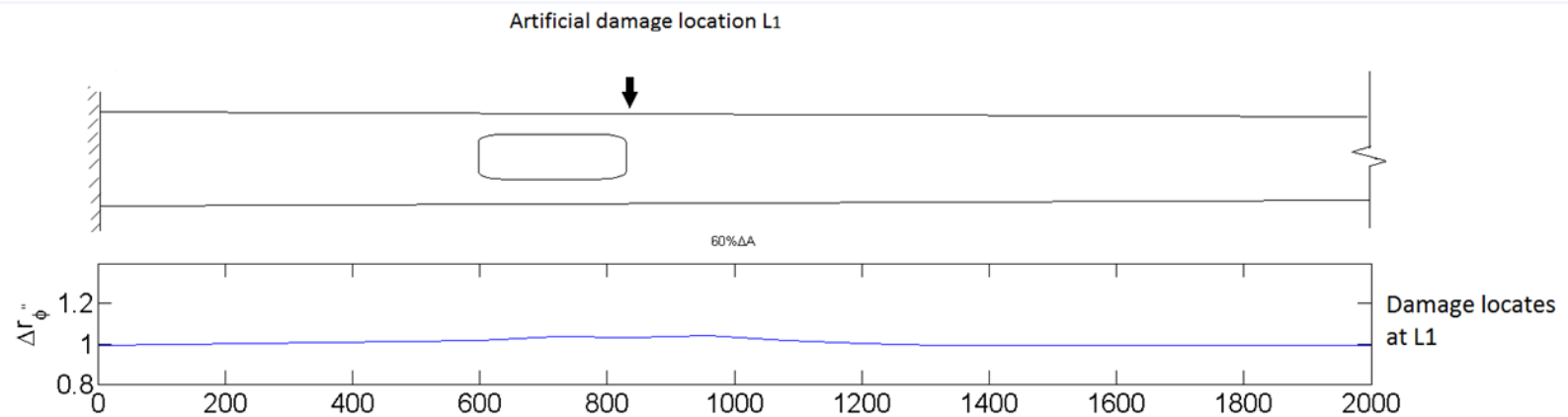
Conclusion

3. Locate damage using mode shape curvature -- In 2nd mode, maximum curvature change ($\Delta r_{\phi''}$) occurs at damage location. Therefore, one can use changes in curvature of 2nd mode shape to localize damages. However, this method is **limited**.

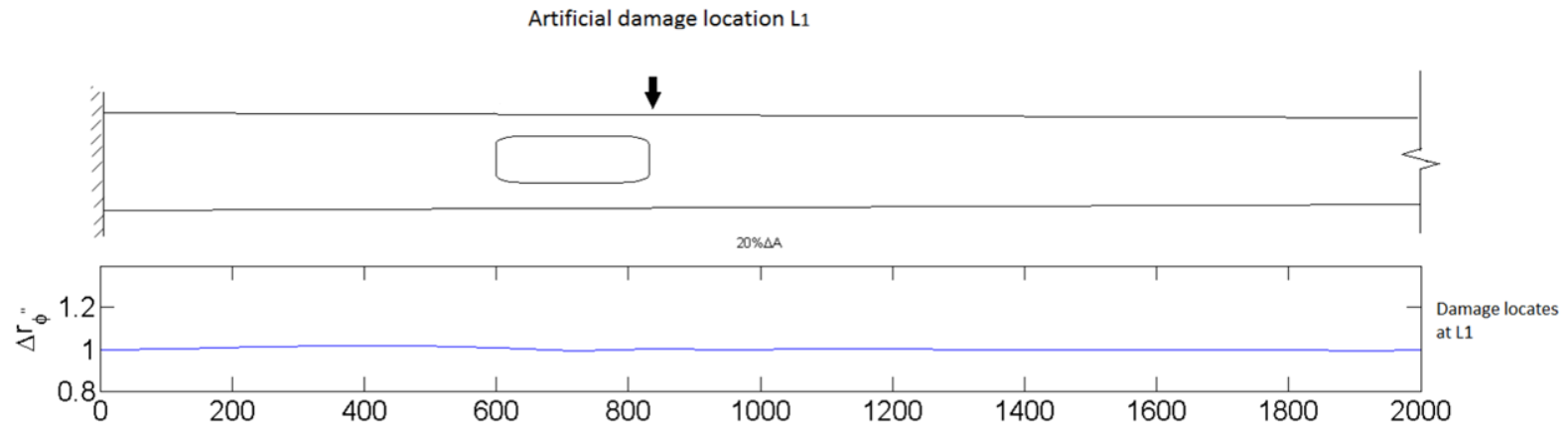
- When damage size is greater than 80% of cross-section area, the maximum curvature changes accurately locate at damage locations.



- When damage size is between 40% to 60% of cross-section area, there will be shifts between the maximum curvature change location and damage location.



- When damage size is lower than 20% of cross-section area, the curvature change is not sensitive to localize the damage.



Conclusion

Future work

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