CIVE.5120 Structural Stability (3-0-3)
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Buckling of Beam-Columns – II

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Outline

- Method of superposition
  - To determine the maximum moment:
- Simply-supported beam-column with a uniformly distributed load and a concentrated load
  - To determine fixed-end moments:
    - Fixed-end beam-columns with a uniformly distributed load
    - Fixed-end beam-columns with a uniformly distributed load and a concentrated load
- Differential equation approach
  - Second-order vs. fourth-order
- Slope-deflection and modified slope-deflection equations
- Inelastic beam-columns
- Summary

Beam-Columns – II

- **Method of superposition for determining the maximum moment**
  – Simply-supported beam-column with a uniformly distributed load and a concentrated load

Beam-Columns – II

- **Method of superposition for determining fixed-end moments**
  - Fixed-end beam-columns with a uniformly distributed load

Beam-Columns – II

- **Method of superposition for determining fixed-end moments**
  – Fixed-end beam-columns with a uniformly distributed load and a concentrated load

Beam-Columns – II

- **Differential equation approach**
  - Second-order vs. fourth order

Beam-Columns – II

- Fixed-end beam-column with **a concentrated load at mid-span**:
  - Governing equation:
  - Maximum internal bending moment: M_{max}

Beam-Columns – II

- Fixed-end beam-column with **uniformly distributed loads**:
  - Governing equation:
  - Maximum internal bending moment: $\boldsymbol{M_{max}}$

Beam-Columns – II

- Fixed-end beam-column with **a concentrated bending moment at midspan:**
  - Governing equation:
  - Maximum internal bending moment: $\boldsymbol{M_{max}}$

Beam-Columns – II

- **Slope-deflection equations**
  - Assumptions:
    - Prismatic beam
    - No relative joint displacement (no sidesway)
    - Continuous member (no internal hinges)
    - No in-span transverse loadings
    - Compressive axial force

Beam-Columns – II

- **Slope-deflection equations**
  - Deflection function for a beam-column
  - Stability functions: $\boldsymbol{S_{ij}}$

Beam-Columns – II

- **Modified slope-deflection equations**
  - Members with **sidesway**

Beam-Columns – II

- **Modified slope-deflection equations**
  - – Members with **rotational springs at the ends**

Beam-Columns – II

- **Modified slope-deflection equations**
  – Members with **transverse loadings**

Beam-Columns – II

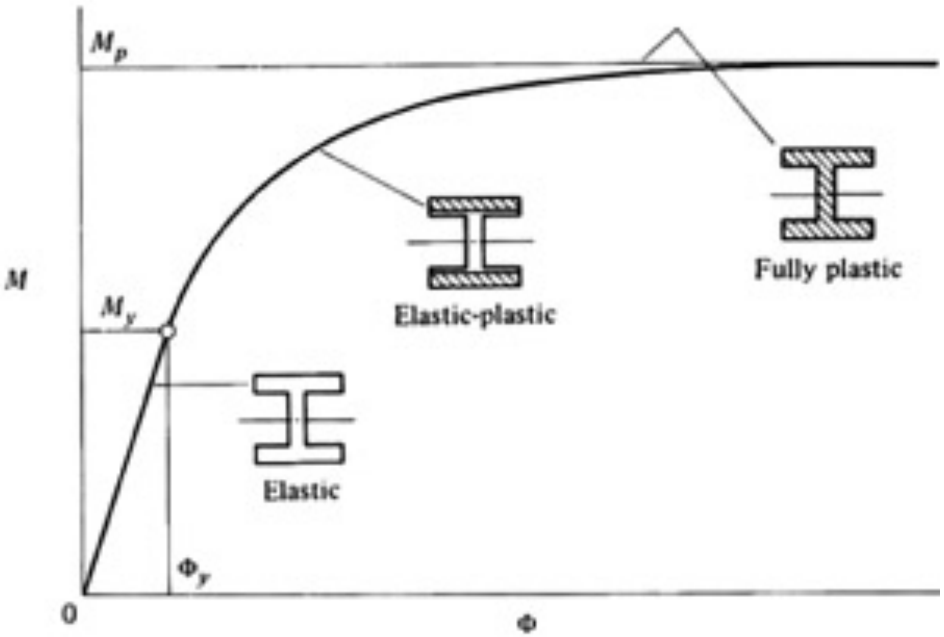
- **Modified slope-deflection equations**
  - Members with **tensile axial force**

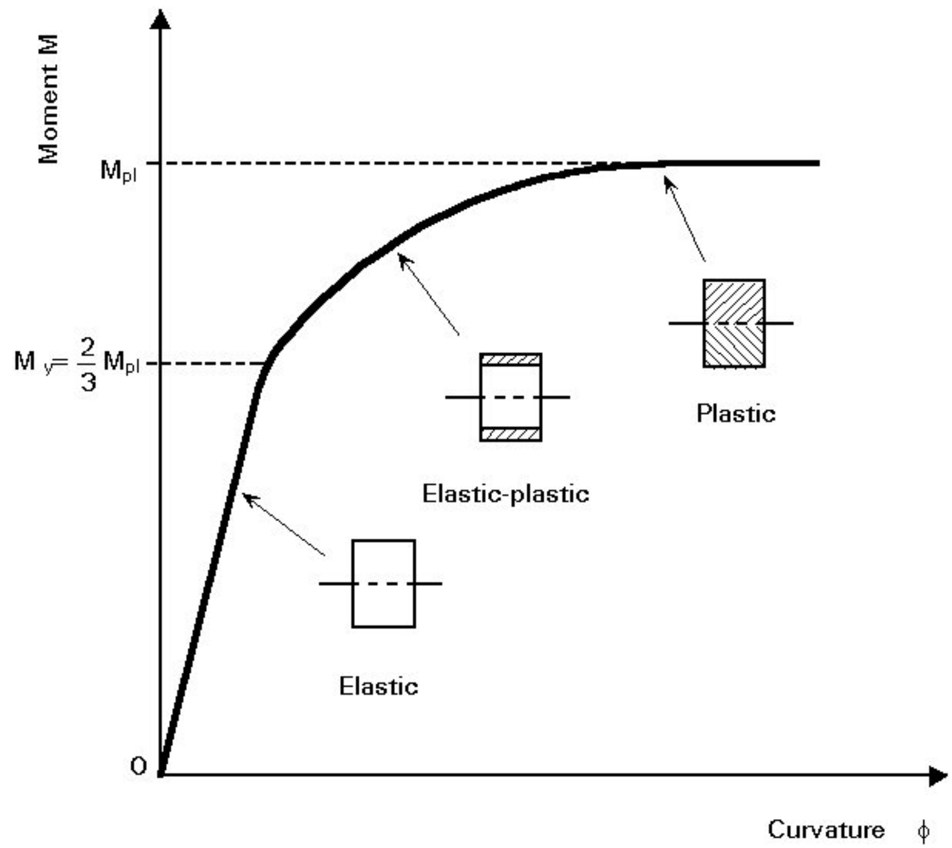
Beam-Columns – II

- **Modified slope-deflection equations**
  - Members bent in **double curvature with $\theta_A = \theta_B$**

Beam-Columns – II

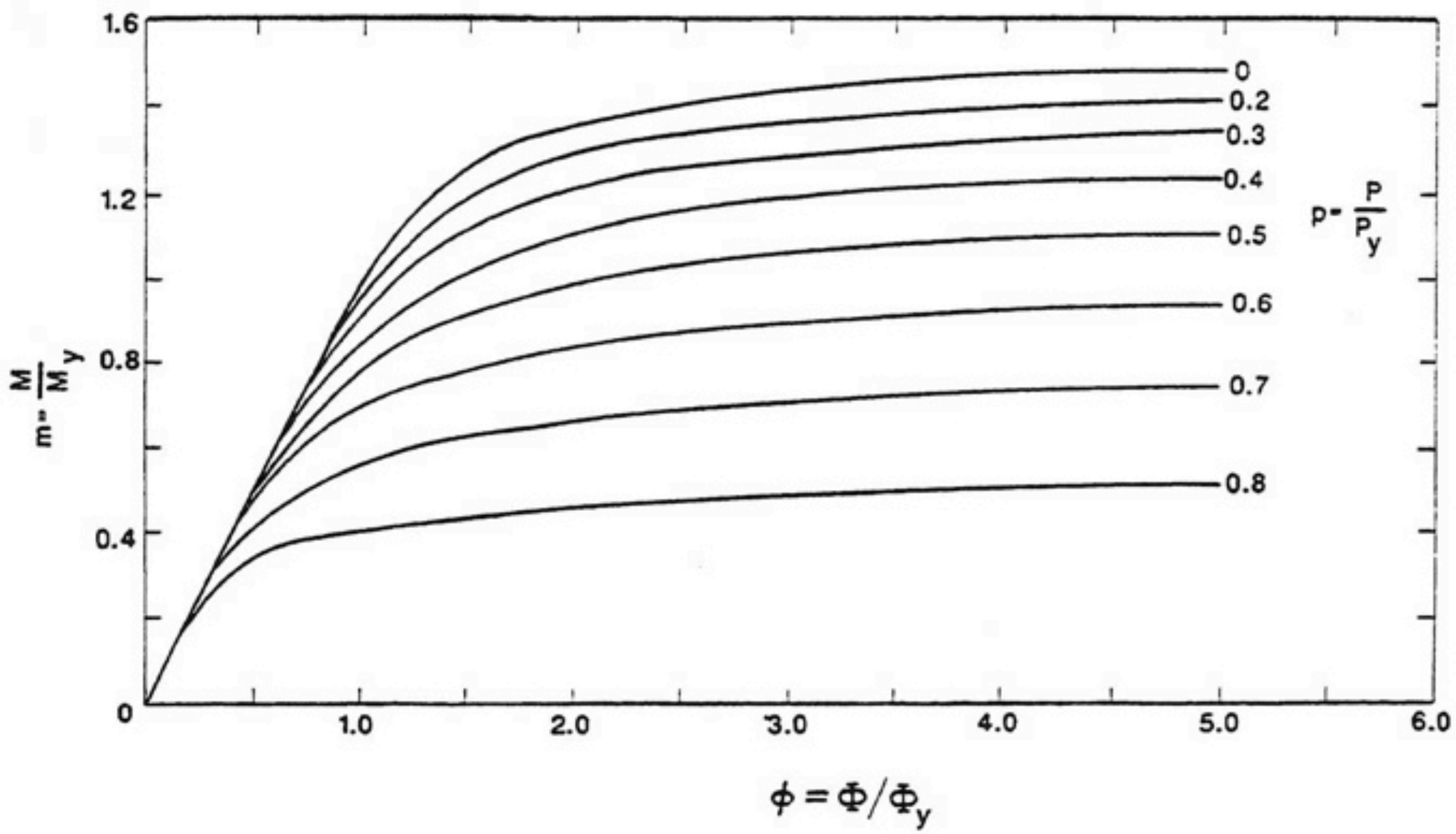
- **Inelastic beam-columns**
  - *$M - \Phi_A$ relationship*





Beam-Columns – II

- **Inelastic beam-columns**
  – *M – Φ_A – P relationship*



Beam-Columns – II

- **Inelastic beam-columns**
  - Case 1: Elastic

Beam-Columns – II

- **Inelastic beam-columns**
  - Case 2: Primary plastic

Beam-Columns – II

- **Inelastic beam-columns**
  - Case 3: Secondary plastic

Summary

- The general governing equation of an elastic beam-column is a fourth-order linear D.E.
- In the region of constant shear force, the use of a third-order linear D.E. is more convenient.
- In the case of an constant external moment, the use of a second-order linear D.E. is more convenient.