Mass and Stiffness Estimation using Mobile Devices for Structural Health Monitoring

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ABSTRACT

In the structural health monitoring (SHM) of civil infrastructure, dynamic methods using mass, damping, and stiffness for characterizing structural health have been a traditional and widely used approach. Changes in these system parameters over time indicate the progress of structural degradation or deterioration. In these methods, capability of predicting system parameters is essential to their success. In this paper, research work on the development of a dynamic SHM method based on perturbation analysis is reported. The concept is to use externally applied mass to perturb an unknown system and measure the natural frequency of the system. Derived theoretical expressions for mass and stiffness prediction are experimentally verified by a building model. Dynamic responses of the building model perturbed by various masses in free vibration were experimentally measured by a mobile device (cell phone) to extract the natural frequency of the building model. Single-degree-of-freedom (SDOF) modeling approach was adopted for the sake of using a cell phone. From the experimental result, it is shown that the percentage error of predicted mass increases when the mass ratio increases, while the percentage error of predicted stiffness decreases when the mass ratio increases. This work also demonstrated the potential use of mobile devices in the health monitoring of civil infrastructure.

Keywords: Structural health monitoring, dynamic response, perturbation analysis, mobile devices

1. INTRODUCTION

Structural health monitoring (SHM) of critical civil infrastructures like buildings and bridges provide valuable information for the governments and civil engineers to maintain these structures. Structural integrity (e.g., stiffness) needs to be monitored such that colossal failures of structures can be prevented. In the SHM applications of civil infrastructure, dynamic approach based on the vibrational behavior of structures has been widely used.[1–3] In this approach, structural properties (mass, damping, and stiffness) are targeted and to be identified for condition assessment. Profound knowledge about the variations of mass, damping and stiffness is crucial to the success of SHM when using dynamic approaches. Change in mass can be used to indicate either material degradation or loss of cross sectional area. Change in damping can be used to indicate the presence of cracks, steel corrosion or the change in connections/joints. Change in stiffness can be used to predict cracking or corrosion. All these structural properties are essential to the definition of structural health in SHM.

Wired and wireless sensor networks have been applied to instrument bridges and buildings for SHM. For example, accelerometers are used to measure acceleration and linear variable differential transformers (LVDTs) to measure displacement. Conversion among acceleration, velocity and displacement can reduce the amount of direct measurements if the constant offset issue can be properly handled in an integration process. Meanwhile, advances in sensing technologies and mobile devices have enabled civil engineers to use the sensors on mobile devices (e.g., smart phones, tablet computers) for data collection, signal processing and data visualization.[4] With such capabilities, on-site inspections of critical civil infrastructures can be much more efficient and effective than before.

The objective of this paper is to present our research work on the use of a mobile device (smart phone) for estimating mass and stiffness of structures using a laboratory building model as an example. A perturbation

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method was developed to extract the unknown mass and stiffness of the building model, using accelerations measured by the smart phone. The building model was perturbed multiple times in free vibration with known perturbation masses. Damping values were also estimated by mathematical optimization using the least squares criterion. Actual mass and stiffness of the building model were independently measured by a precision scale (for mass) and a digital hanging scale (for stiffness) for validation. In all experiments, only one smart phone was used.

In this paper, theoretical basis of the perturbation method is first introduced. Experimental configuration of the building model and smart phone instrumentation are described. Processed accelerations and estimated mass and stiffness are reported in the results section. Finally, research findings and issues are summarized.

2. THEORETICAL BACKGROUND

2.1 Mass perturbation

Consider an under-critically damped single degree of freedom (SDOF) system. Its natural frequency can be expressed by

$$\omega_d = \sqrt{(1-\xi)} \cdot \omega_n = \sqrt{(1-\xi) \cdot \frac{k}{m}} \tag{1}$$

where ω_d is the damped natural frequency, ξ the damping ratio, ω_n the undamped natural frequency, k the stiffness and m the mass of the SDOF system. Perturb the SDOF system by introducing a perturbation mass (Δm) to the system. The damped natural frequency of a perturbed SDOF system can written by

$$\omega_{d1} = \sqrt{(1-\xi_1) \cdot \frac{k_1}{m_1}} = \sqrt{(1-\xi_1) \cdot \frac{k_1}{m+\Delta m_1}} \tag{2}$$

$$\omega_{d2} = \sqrt{(1-\xi_2) \cdot \frac{k_2}{m_2}} = \sqrt{(1-\xi_2) \cdot \frac{k_2}{m+\Delta m_2}} \tag{3}$$

where ω_{di}, ω_{ni}, ξ_i, Δm_i are the damped natural frequency, undamped natural frequency, damping ratio, and perturbation mass of perturbed SDOF system in the i^{th} perturbation, respectively. The mass condition leads to

$$m = \left(\frac{1-\xi_1^2}{\omega_{d1}^2}\right) k_1 - \Delta m_1 = \left(\frac{1-\xi_2^2}{\omega_{d2}^2}\right) k_2 - \Delta m_2 \tag{4}$$

Or

$$m = \frac{k_1}{\omega_{n1}^2} - \Delta m_1 = \frac{k_2}{\omega_{n2}^2} - \Delta m_2 \tag{5}$$

In theory, estimation of mass using Eq.(5) is simple if all three other terms can be determined with confidence. Meanwhile, from Eq.(5), we have

$$k_2 = \omega_{n2}^2 \cdot \left(\frac{k_1}{\omega_{n1}^2} - \Delta m_1 + \Delta m_2\right) \tag{6}$$

or generally,

$$k_i = \omega_{ni}^2 \cdot \left(\frac{k_j}{\omega_{nj}^2} - \Delta m_j + \Delta m_i\right) \tag{7}$$

in which i and j indicate two different perturbations. For intact structures, $k_j = k$, $\omega_{nj} = \omega_n$ and $\Delta m_j = 0$. In this paper, it is of interest to investigate the practical performance of Eqs.(5)-(7) using a building model.

2.2 Optimization using theoretical acceleration

In the second part of the proposed method, optimal combination of undamped natural frequency (ω_n) and damping ratio (ξ) is determined by comparing experimental accelerations with theoretical values. For under-critically damped SDOF systems in free vibration, their displacement function is described by the following:[5]

$$u(t) = \exp(-\xi\omega_n t)\left[u(0)\cos(\omega_d t) + \frac{\dot{u}(0) + \xi\omega_n u(0)}{\omega_d}\sin(\omega_d t)\right] \tag{8}$$

where t is the time variable, $u(0)$ the initial displacement and $\dot{u}(0)$ the initial velocity. The second-order derivative of $u(t)$ leads to $\ddot{u}(t) = a(t)$, which is

$$\begin{aligned}\ddot{u}(t) = (\xi\omega_n)^2\exp(-\xi\omega_n t)\cdot&\left[u(0)\cos(\omega_d t) + \frac{\dot{u}(0) + \xi\omega_n u(0)}{\omega_d}\sin(\omega_d t)\right]\\ -\xi\omega_n)\exp(-\xi\omega_n t)\cdot&\left[-\omega_d u(0)\sin(\omega_d t) + (\xi\omega_d u(0))\right]\\ +\exp(-\xi\omega_n t)\cdot&\left[-u(0)\omega_d^2\cos(\omega_d t) - \xi u(0)\omega_d^2\sin(\omega_d t)\right]\end{aligned} \tag{9}$$

In a SHM problem using Eq.(9), the goal is to determine structural properties (system parameters); ω_n and ξ. ω_d can be found by $\omega_d = \omega_n\sqrt{1-\xi^2}$ for under-critically damped SDOF systems. To determine the optimal combination of ω_n and ξ, theoretical acceleration $\ddot{u}_{theo}(t)$ (using Eq.(9)) and and experimental acceleration $\ddot{u}_{exp}(t)$ (using the smart phone sensor) were defined. The least squares criterion was used to determine the optimal combination of ω_n and ξ in order to result in a minimum error between $\ddot{u}_{theo}(t)$ and $\ddot{u}_{exp}(t)$.

3. EXPERIMENTAL SETUP

3.1 Smart phone sensor

In the experimental testing of the proposed mass and stiffness estimation method, a smart phone (LG Nexus 5®by LG Electronics, Android 5.0 operation system) was used as an accelerometer. Mobile application *Accelerometer Monitor* was used to record triaxial accelerations at a 200-Hz sampling rate. The mass of the smart phone was measured by a precision scale and found to be 0.17 kg. The smart phone sensor was calibrated by attaching it to a digital function generator (PASCO®PI-8127, 0.001 Hz to 150 Hz, 10 Volts at 1 Amp) and a mechanical vibrator (PASCO®SF-9324) (Fig. 1). Accelerations measured by the smart phone sensor were compared with input functions in time domain, as well as in frequency domain by Fast Fourier Transform (FFT). From the calibration, it was found that the smart phone sensor (accelerometer) precisely measures the excitation frequency produced by the function generator.

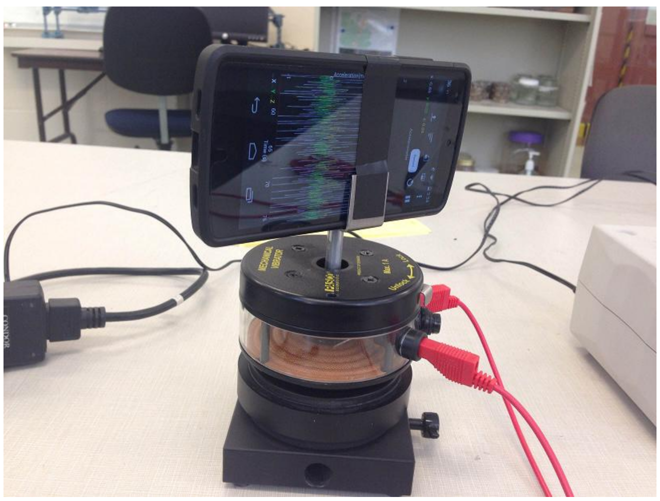

Figure 1. Calibration of the smart phone sensor using a mechanical vibrator

Table 1. Mass perturbation cases

Case	Base mass (kg)
A (1 top plate)	1.3024
B (5 added plates)	2.2571
C (5 added plates and 6 blocks)	3.5684
D (5 added plates and 3 kg weight)	5.2571

3.2 Building model

A five-story building frame model was constructed for this research. In Fig. 2, the building frame model was assembled by plastic elements and steel screws. Specific joint elements were available to ensure a fixed boundary condition between beams and columns. The model was attached to a base plate to simulate a fixed support boundary condition. The total mass of the model was measured by the scale and found to be 1.3024 kg. The stiffness of the model in a cantilever model was measured by a digital hanging scale and found to be 5,182.9 N/m. This building model is an under-critically damped system, which was demonstrated in acceleration measurements.

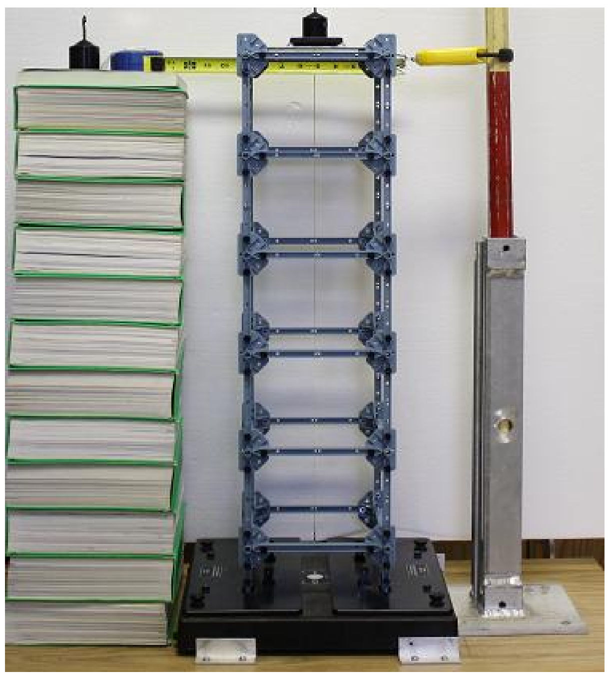

Figure 2. A five-story building model

3.3 Mass perturbed free vibration

Four mass perturbation cases considered in this research are listed in Table 1. There are four perturbation masses; 0.17 kg (smart phone), 0.37 kg (phone plus 0.2 kg weight), 0.67 kg (phone plus 0.5 kg weight) and 1.17 kg (phone plus 1 kg weight). These cases are also shown in Fig. 3. In all cases, the smart phone sensor is always instrumented on the top floor of the building model. Damped natural frequencies (ω_d) of the building model perturbed by various masses were measured in free vibration with the smart phone sensor. The smart phone sensor was instrumented on the top floor of the building model, and it recorded the oscillation of the model triggered by an initial displacement of 0.25 in (0.635 cm). Fig. 4 shows an example acceleration curve of the building model. From the frequency spectrum, the damped natural frequency (ω_d) can be found. For instance, $\omega_d = 2\pi(8.5938)$ rad-Hz in Fig. 4.

(a) Case A *(b) Case B* *(c) Case C* *(d) Case D*

Figure 3. Four mass perturbation cases

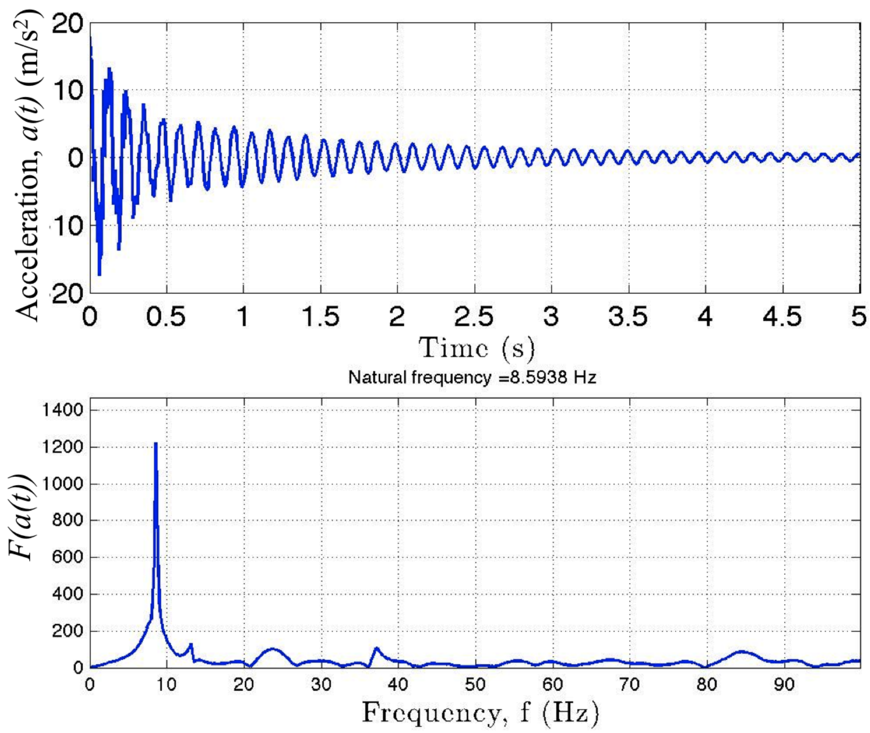

Figure 4. Example acceleration $a(t)$ measured by the smart phone sensor and its frequency spectrum; Case A1, initial displacement = 0.25 in.

4. RESULT AND ANALYSIS

4.1 Experimental acceleration

Accelerations of building models perturbed by various masses were collected by the smart phone sensor. Fig. 5 shows the acceleration curves of the building model perturbed by 0.17 kg mass, 0.37 kg mass, 0.67 kg mass and 1.17 kg mass as an example for three other cases.

Table 2. Optimal combination of ω_n and ξ in all cases

Case	Perturbed mass (kg)	ω_n (Hz)	ξ
A1	0.17	8.563	0.025
A2	0.37	7.735	0.04
A3	0.67	6.350	0.05
A4	1.17	5.682	0.025
B1	0.17	6.239	0.04
B2	0.37	6.987	0.025
B3	0.67	5.730	0.025
B4	1.17	5.379	0.025
C1	0.17	4.631	0.02
C2	0.37	5.825	0.02
C3	0.67	4.822	0.045
C4	1.17	4.584	0.04
D1	0.17	5.332	0.025
D2	0.37	5.141	0.025
D3	0.67	4.393	0.035
D4	1.17	4.058	0.035

4.2 Mathematical optimization

Using the least squares error criterion, performance of different combinations of ω_n and ξ can be quantified. Ranges of optimization on ω_n and ξ were [30, 50] (rad-Hz) (or [4.7746, 7.9577] (Hz)) and [0, 0.1], respectively. Table 2 lists the optimal combinations of ω_n and ξ for all cases. All optimization curves contain single value minimum (error).

Fig. 6 illustrates the performance of the optimal (ω_n, ξ), while Fig. 7 shows the performance of all combinations. In Fig. 7, it is clear that the combination of (5.332 Hz, 0.025) in Case D1 produces a minimum error. All other optimal combinations of ω_n and ξ in Cases A to D were determined in the same approach.

4.3 Predicted Mass and Stiffness

Using Eqs.(5)-(7), mass and stiffness are predicted by acceleration measurements in free vibration. These predicted values are compared with independently measured mass (Table 3) and stiffness (Table 4. In Tables 3 and 4, mass ratio (r_m) is define by

$$r_m = \frac{m_p}{m_b} \tag{10}$$

where m_p is the perturbation mass and m_b the base mass. Errors in mass and stiffness prediction are defined in the following.

$$Error_m(\%) = \frac{m_{theo} - m_{actual}}{m_{actual}} \times 100\% \tag{11}$$

$$Error_k(\%) = \frac{k_{theo} - k_{actual}}{k_{actual}} \times 100\% \tag{12}$$

where m_{theo} and k_{theo} are predicted mass and stiffness, respectively. m_{actual} and k_{actual} are independently measured mass and stiffness, used as the ground truth for validating the proposed method. After plotting the relationships between prediction errors and mass ratio for mass and stiffness in Figures 8 and 9, it is found that the prediction error for mass increases with the increase of mass ratio, while the prediction error for stiffness decreases with the increase of mass ratio. In other words, there exists an optimal mass ratio when minimum prediction errors are required for both mass and stiffness.

Table 3. Measured and predicted mass values in all cases

Case	Mass ratio (r_m)	Measured mass (kg)	Predicted mass (kg)
A1	0.131	1.473	1.621
A2	0.284	1.673	1.752
A3	0.515	1.973	2.474
A4	0.898	2.473	2.746
B1	0.075	2.427	3.156
B2	0.164	2.627	2.198
B3	0.297	2.927	3.038
B4	0.518	3.427	2.975
C1	0.048	3.738	5.700
C2	0.104	3.938	3.279
C3	0.188	4.239	4.421
C4	0.328	4.738	4.438
D1	0.032	5.427	4.158
D2	0.070	5.627	4.223
D3	0.223	5.927	5.436
D4	0.127	6.427	5.841

Table 4. Measured and predicted stiffness values in all cases

Case	Mass ratio (r_m)	Measured stiffness (N/m)	Predicted stiffness (N/m)
A1	0.131	5,182.9	4,701.8
A2	0.284	5,011.9	3,723.7
A3	0.515	5,005.8	3,556.6
A4	0.898	4,991.4	5,438.9
B1	0.075	5,110.8	6,795.3
B2	0.164	4,950.2	3,848.4
B3	0.297	4,806.0	4,942.1
B4	0.518	4,736.0	4,724.6
C1	0.048	4,970.8	8,131.2
C2	0.104	4,888.4	3,690.4
C3	0.188	4,674.6	5,698.3
C4	0.328	4,674.2	4,689.7
D1	0.032	4,875.5	4,724.4
D2	0.070	4,791.6	3,506.7
D3	0.223	4,651.6	3,464.8
D4	0.127	4,558.9	4,959.8

5. SUMMARY AND DISCUSSION

From the experimental result, research findings are summarized in the following.

- Acceleration measurement using a smart phone sensor – In this research, it is found that mobile devices like smart phones are capable of capturing the natural frequency of SDOF systems in the frequency range of [4.058, 8.563] Hz. While mobile applications like *Acceleration Monitor* was not used for analyzing the frequency content of accelerations, it is apparent that future mobile applications can perform advanced signal processing (e.g., FFT) and even conduct SHM in the field on civil infrastructures like buildings and bridges. Fig. 5 shows four example acceleration measurements in Case A.
- Mass and stiffness prediction – In theory, Eqs.(5)-(7) are capable of predicting mass and stiffness in a mass-perturbed SDOF system. Nonetheless, from the obtained experimental result, the theory does not predict its experimental counterpart. Possible reasons include:
  1. Accuracy of theoretical models – The use of an under-critically damped SDOF system cannot capture all dynamic characteristics of continuous systems like the building model, even though simple models (e.g., DOF, boundary conditions, loading function) still represent a practical value for preliminary and efficient SHM. The trade-offs between simple and comprehensive models are well known to the SHM community.
  2. Mass ratio – In this paper, the main reason for choosing mass perturbation (as oppose to stiffness perturbation) is due to the ease of attaching mass to actual structures in practice. Such approach is instrumented by the use a quantitative index, mass ratio or r_m, to evaluate the performance of mass and stiffness predictions. With this ratio, it is experimentally found that there is a ascending trend between the mass ratio and the error in mass prediction and a descending trend between the mass ratio and the error in stiffness prediction. This suggests the existence of an optimal mass ratio for achieving best mass and stiffness prediction at the same time.
  3. Limitations of single acceleration – Since only one smart phone sensor was used in this paper, other dynamic parameters such as mode shapes cannot be measured and used in the mass and stiffness prediction of this research. Should more information (e.g., mode shapes) can be provided, more accurate predictions can be achieved in a noisy
  4. Effect of background noise – In this paper, denoising methods like bandpass filters were not used, neither were all kinds of background noise modeled in the mass and stiffness prediction. Better predictions can be expected if the measured accelerations are denoised.

6. CONCLUSION

In this paper, a mass and stiffness prediction method using a commercially available smart phone sensor is proposed. Feasibility of the proposed method is demonstrated by the use of a building model. Accelerations of a building model perturbed by various masses attached to the building model can be measured by the smart phone sensor. It is experimentally found that there is a ascending trend between the mass ratio and the error in mass prediction and a descending trend between the mass ratio and the error in stiffness prediction, suggesting the existence of an optimal mass ratio for achieving best mass and stiffness prediction at the same time. From this research, it is believed that the use of smart phones for in-situ SHM of critical civil infrastructures is of great potential.

7. ACKNOWLEDGMENT

The authors want to thank the National Institute of Standards and Technology (NIST) Technology Innovation Program (TIP) for partially supporting this research. The author (Le) also thanks the American Society for Nondestructive Testing (ASNT) for providing the financial support through the 2014 ASNT Engineering Undergraduate Scholarship.

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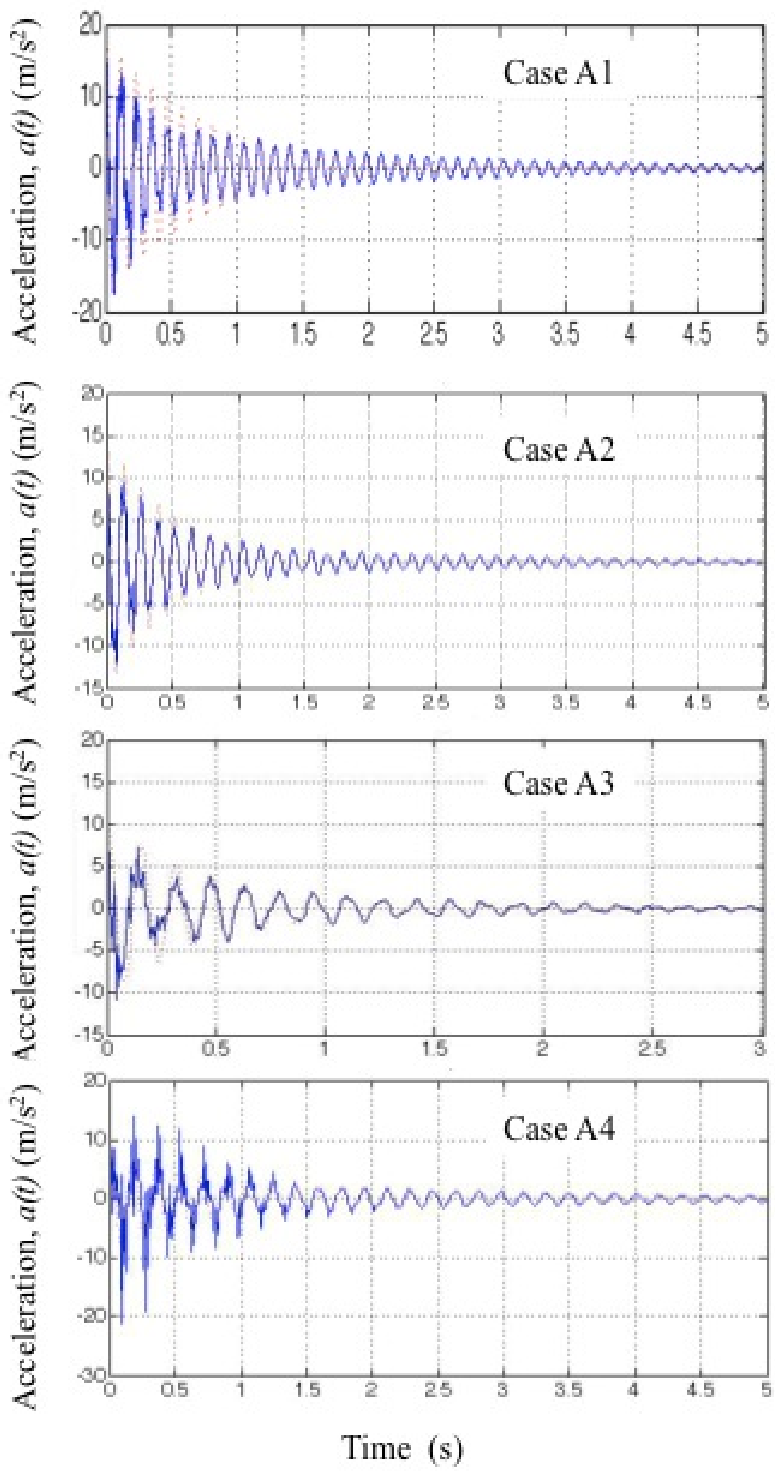

Figure 5. Acceleration curves for Case A1 A4

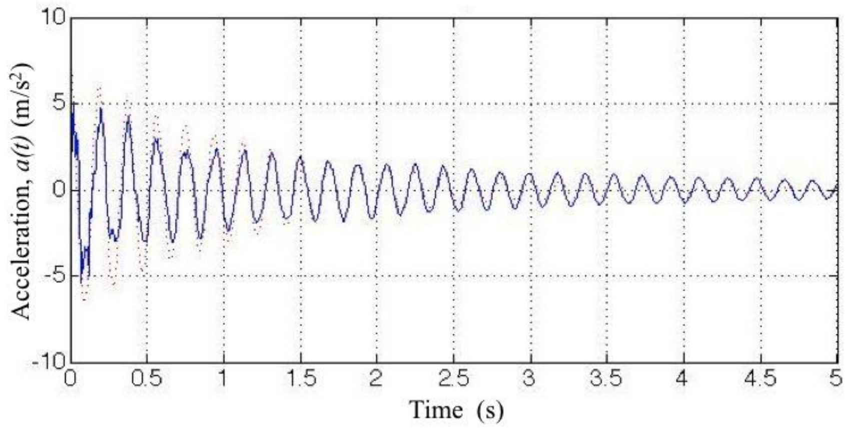

Figure 6. Performance of the optimal combination of (ω_n, ξ) in Case D1, perturbed mass = 0.17 kg; $(\omega_n, \xi)_{opt}$ = (5.332 Hz, 0.025) / solid line: experimental acceleration; dashed line: predicted acceleration with optimal $(\omega_n, \xi)_{opt}$

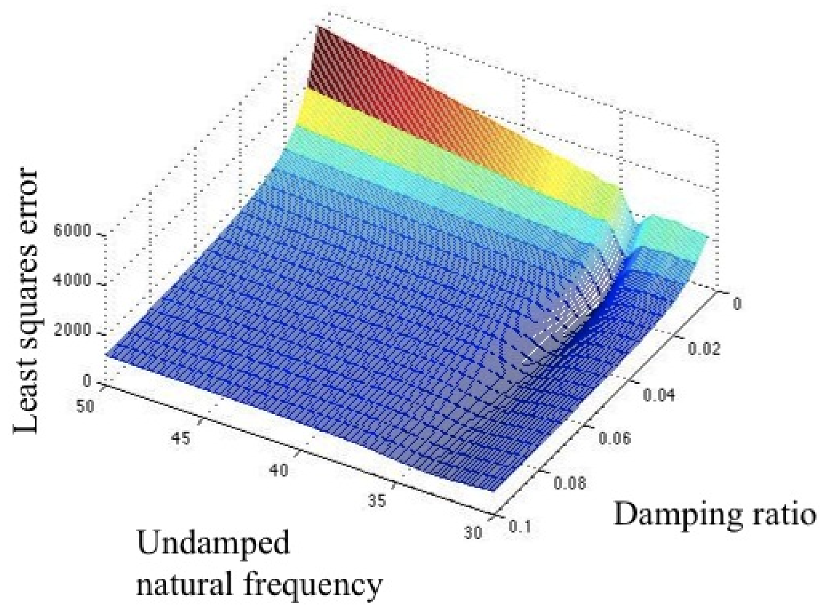

Figure 7. Performance of all combinations of (ω_n, ξ) in Case D1, perturbed mass = 0.17 kg

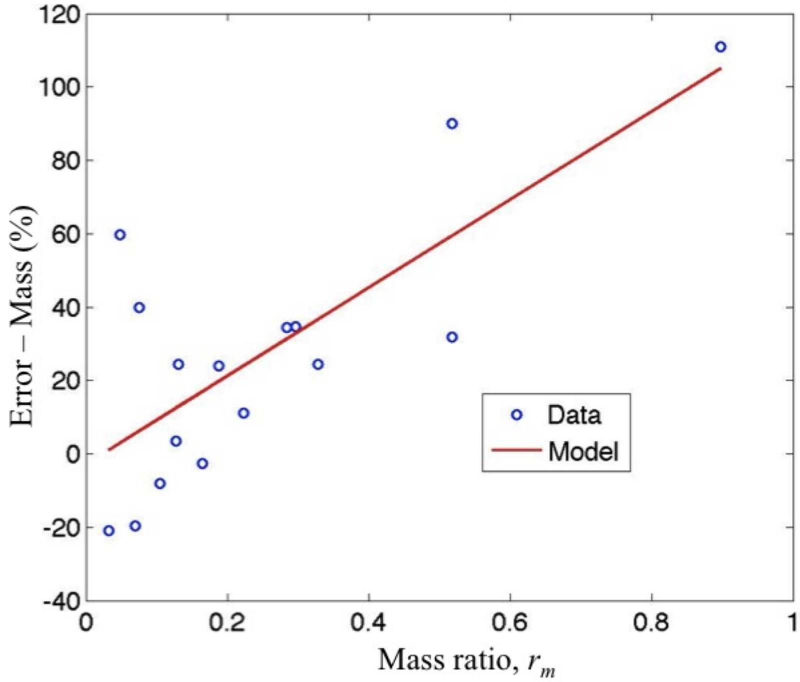

Figure 8. Prediction error vs. mass ratio – Mass

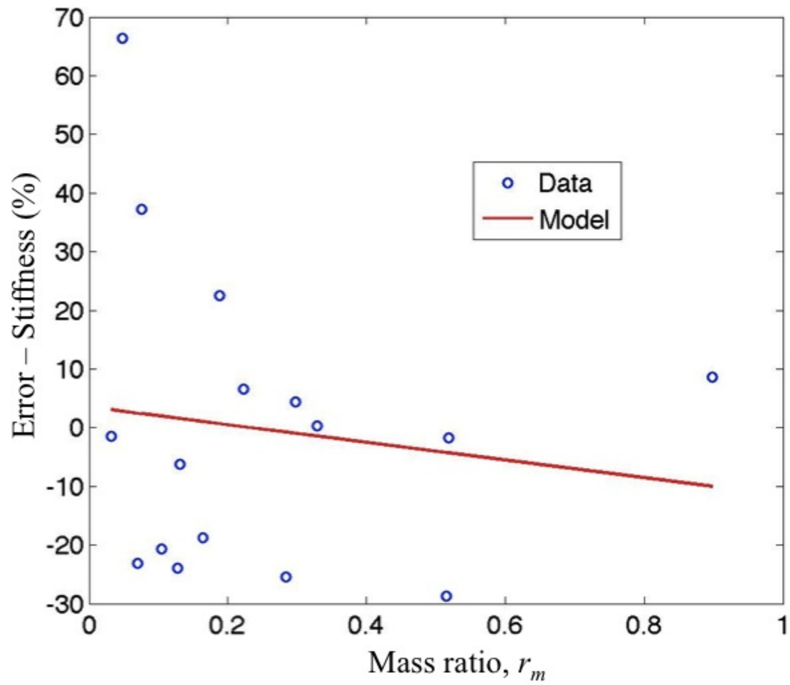

Figure 9. Prediction error vs. mass ratio – Stiffness